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HIGHWAY FILL CONSTRUCTION OPERATIONS

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The reports of research published in this magazine are necessarily qualified by the conditions of the tests from which the data are obtained. Whenever it is deemed possible to do so, generalizations are drawn from the results of the tests; and, unless this is done, the conclusions formulated must be considered as specifically pertinent only to described conditions.

In This Issue

	Page
The Theory of Soil Consolidation and Testing of Foundation Soils	1

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THE THEORY OF SOIL CONSOLIDATION AND TESTING OF FOUNDATION SOILS

BY THE DIVISION OF TESTS, BUREAU OF PUBLIC ROADS

Reported by L. A. PALMER, Associate Chemist, and E. S. BARBER, Junior Highway Engineer

SEVERAL DEVICES used in performing the compression test described in this report were constructed by the Bureau of Public Roads in 1926 in accordance with plans furnished by Dr. Charles Terzaghi. Investigation of testing technique and practical utilization of the resulting data have been included in the Bureau's program of research on soils for highway purposes continuously since that time.

As part of this program, the Bureau cooperated with the Massachusetts Institute of Technology in research on soil mechanics from 1926 until the fall of 1932. Dr. Terzaghi was retained by the Bureau as research consultant on the work from the fall of 1926 to the fall of 1929. Dr. Arthur Casagrande was employed by the Bureau as research assistant and was stationed at the Massachusetts Institute of Technology throughout the duration of this cooperative research work.

Other parts of the Bureau's general program were carried on at the Bureau's laboratory at Arlington, Va., and through cooperation with various State highway departments and Federal bureaus.

Special studies have been made at Arlington to determine the magnitude of experimental errors resulting from variables in testing apparatus and procedure, and to determine the agreement between deformations observed in tests with those computed in accordance with the theory of consolidation on which the tests are based.

The compression test and theory of soil consolidation were described by Terzaghi in 1927.¹ Two other reports of distinctive value to those concerned with the practical utilization of the compression test data have also been published.²

On the basis of the Bureau's investigations of the test itself, experience in practical use of the test data, and reports just referred to, it is now possible (1) to state definitely that the apparatus and testing procedure described briefly in this report have been found satisfactory, and (2) to present a practical method of estimating that part of the total settlement of soil caused by the loss of water forced vertically out of saturated compressible soil strata in certain soil profiles.

Settlements of soils in profiles differing from those specifically described, or produced by plastic flow, flow of water laterally, or causes not discussed, are beyond the scope of this presentation.

COMPRESSION DEVICE VALUABLE AID IN STUDYING SOIL COMPRESSION

The theory of consolidation of earth materials involves two problems: (1) Determination of the distribution of stresses; and (2) computation of the displacements caused by these stresses. In dealing with many engineering materials and structures the essential

problem is to estimate the stresses, since nearly all structural materials possess, within limits, the property of elasticity. With soils, however, the problem is quite complicated for they have both plastic and elastic properties; the materials are essentially heterogeneous; and water is usually present. If the water is free to flow from the compressed soil most of the settlement results directly from the reduction in voids between solid particles, an occurrence that, in this paper, is considered apart from elastic and plastic deformations.

One of the most useful tools in the study of soil consolidation is the laboratory compression device designed by Dr. Terzaghi. The compression tests indicate within a comparatively short time in the laboratory the consolidation characteristics of soils upon which depend the rates of settlement and total settlements of foundations. Thus, compression tests supplant guesswork and "rule of thumb" methods with reliable information for use in the design of foundations.

Figure 1 illustrates the essential features of the testing device used by the Bureau of Public Roads. An undisturbed sample of soil with natural structure and moisture content, as nearly as can be obtained, is carefully placed between two porous stones, which act as filters during the test. Pressures are then applied, ranging from zero to the maximums that are expected in service. A burette connected with the apparatus as shown in figure 1 affords a means of determining the permeability of the sample at any time during the test.

The sample during test is representative of a stratum of saturated, compressible soil in the earth's crust, sandwiched between two layers of sand or other relatively permeable material.

The total amount of consolidation and the speed at which it takes place are controlled by: (1) The moisture content of the soil; (2) the velocity with which water flows vertically through the pores of the sample to the filters above and below in the laboratory, or through the compressible layer to the sand strata in the field; and (3) the frictional resistance of the soil particles to consolidation.

At the instant of load application, the water in the compressible sample or layer is considered as furnishing all of the resistance to consolidation. Thus, if a suitable pressure gauge were connected with the sample in the laboratory, it would show that, at the instant of loading, the water pressure in the sample is practically equal to the pressure applied to the sample. It was stated during the first conference on soil mechanics and foundation engineering, held at Harvard University in June, 1936, that experiments in Sweden disclosed that the water pressures in compressible understrata were approximately equal to the pressures produced by the weight of superimposed fill materials just at the time construction of the fill was completed.

As the pressure on the soil continues, water escapes from the sample or field stratum, causing the pressure to be gradually transferred to the soil particles or

¹ Principles of Final Soil Classification, PUBLIC ROADS, vol. 8, no. 3, May 1927.
² Report of the Special Committee on Earths and Foundations, Proceedings of the American Society of Civil Engineers, vol. 59, no. 5, May 1933, and discussion by William P. Kimball, Proceedings of the American Society of Civil Engineers, vol. 59, no. 6, August 1933.

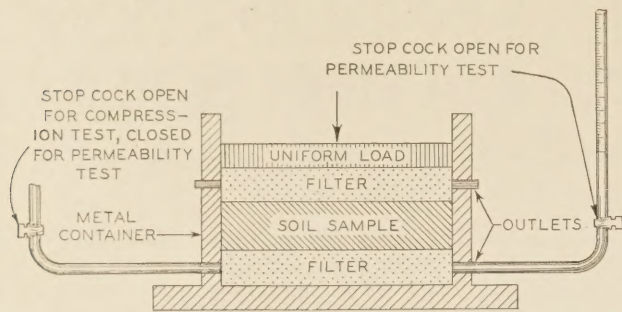


FIGURE 1.—DEVICE FOR DETERMINING THE CONSOLIDATION CHARACTERISTICS OF SOILS IN THE LABORATORY.

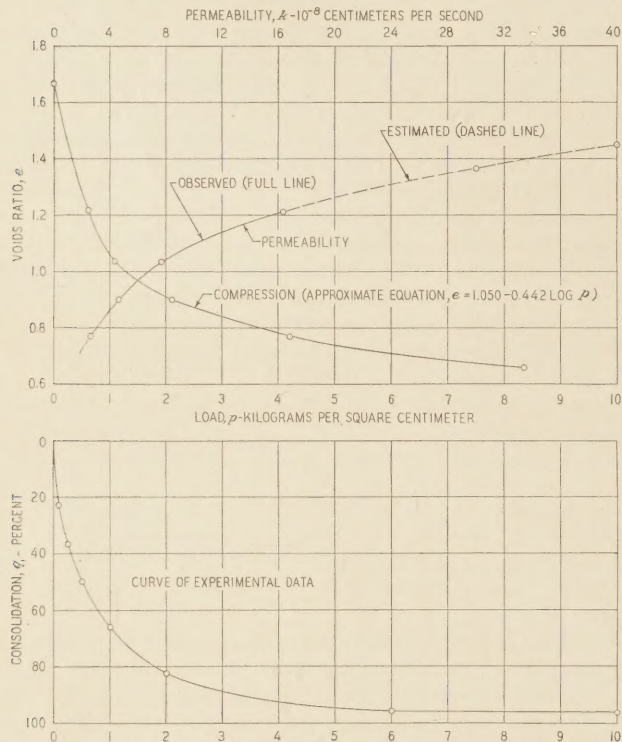


FIGURE 2.—PERMEABILITY, COMPRESSION, AND CONSOLIDATION CURVES FOR A SAMPLE OF COMPRESSIBLE SOIL.

“soil skeleton.” Eventually, equilibrium is reached and further consolidation ceases when the water pressure becomes equal to zero and the soil skeleton provides all of the resistance to consolidation.

The sample in the compression device is subjected to pressure that is increased by increments. Each increment of pressure acts until apparent equilibrium is attained before the next increment is added. Water escapes under pressure, entering the porous stones and flowing from the outlets as indicated in figure 1.

Consolidation characteristics and permeability of the laboratory specimen are recorded and the data are plotted as shown in figure 2. The load-compression curve shows the voids ratios at which the soil sample attained apparent equilibrium under different pressures. The voids ratio is defined as the ratio (by volume) of voids to solids for a given sample of soil.

Apparent equilibrium in the sample is reached in a relatively short time, and for all practical purposes is true equilibrium. Actually, however, the attainment of true equilibrium would require infinite time. For

values of the pressure, p , greater than about one-tenth kilogram per square centimeter, the load-compression curve, as shown, is expressed by the equation,³

$$e = 1.050 - 0.442 \log p \text{-----} (1)$$

where e is the voids ratio. The permeability-voids ratio curve (fig. 2, upper) shows the coefficient of permeability of the sample at different stages of compression.

COEFFICIENTS OF PERMEABILITY AND COMPRESSIBILITY DETERMINED

The coefficient of permeability is defined as the velocity of flow when the hydraulic gradient is unity, that is, when the drop in hydrostatic pressure in grams per square centimeter between two points in a soil column equals the distance in centimeters between the points. The usual practice is to record and plot the coefficient of permeability in centimeters per second, as in figure 2. However, for the purpose of simplifying subsequent computations the coefficient of permeability, as used in this report, is expressed in centimeters per minute. The average time-consolidation curve (fig. 2, lower) shows the rate of consolidation under any given pressure. This curve is the average of the individual time-consolidation curves for different load increments. Essential features of the test procedure, the equation for determining the coefficient of permeability, and the use of average time-consolidation curves have been discussed previously.⁴

Both the consolidation characteristics of the soil strata beneath a given footing and the vertical pressures at various depths produced by the superimposed load must be known. In order to solve the many different problems involved in computing earth pressures, one must be familiar with the mathematical theory applicable to all types of problems requiring the calculation of earth stresses and their distribution. The treatment of special cases is then simplified.

The coefficient of permeability, k , is the velocity of flow at a hydraulic gradient of unity. If the thickness of the soil sample, with a voids ratio of e , is d , and if the voids ratio could be reduced to zero, the thickness would become d_0 , that is,

$$d = d_0(1 + e) \text{-----} (2)$$

The rate at which water would permeate the soil sample if the difference between the two water levels were d_0 , rather than d , is designated as k_0 . It has been shown¹ that the rate of permeation increases as the hydraulic gradient increases so that from this fact and equation (2),

$$\frac{k}{k_0} = \frac{d}{d_0} = 1 + e \text{-----} (3)$$

that is,

$$k_0 = \frac{k}{1 + e} \text{-----} (4)$$

¹ Principles of Final Soil Classification, by Dr. Charles Terzaghi, PUBLIC ROADS, vol. 8, no. 3, May 1927.

² When plotted to a semilogarithmic scale, e being plotted to the natural scale, the load-compression curve is generally a straight line for loads greater than 0.1 kilogram per square centimeter. The general equation for the straight-line portion of the curve is

$$e = B - Z \log p$$

where B is the intercept of the straight-line portion of the curve at the 1.0-kilogram-per-square-centimeter ordinate and $-Z$ is the slope of the semilog plot of the curve, or range in value of e for unit change of $\log p$.

⁴ A Method of Predicting Settlement of Fills Placed on Muck Beds, by F. A. Robeson, PUBLIC ROADS, vol. 16, no. 12, February 1936.

It is convenient to take e as the average voids ratio of a soil sample compressed from an initial voids ratio, e_1 , to an equilibrium value e_2 , so that

$$e = \frac{e_1 + e_2}{2} \text{----- (5)}$$

On substitution of this value for e in equation (4), one obtains

$$k_0 = \frac{2k}{2 + e_1 + e_2} \text{----- (6)}$$

The coefficient of compressibility, a , is the average decrease in voids ratio per unit increase of pressure. If the initial voids ratio, e_1 , is the condition at equilibrium under a pressure of p_1 grams per square centimeter and e_2 is the corresponding condition at a pressure p_2 grams per square centimeter, then

$$a = \frac{e_1 - e_2}{p_2 - p_1} \text{----- (7)}$$

The values of both a and k decrease with the voids ratio, but at different rates. However, the change in the value of the ratio, $\frac{k}{a}$, is fairly small. The ratio, $\frac{k_0}{a}$, is called the coefficient of consolidation, c . From equations (6) and (7) there results from substitution for k_0 and a ,

$$c = \frac{k_0}{a} = \frac{p_2 - p_1}{e_1 - e_2} \times \frac{2k}{2 + e_1 + e_2} \text{----- (8)}$$

The utility of this equation may be illustrated by use of the data plotted in the upper curves of figure 2.

From this curve, the pressures, p_1 and p_2 , corresponding to $e_1 = 1.00$ and $e_2 = 0.80$, are 1,300 and 3,680 grams per square centimeter, respectively. The average voids ratio is 0.90 and from the permeability curve, the coefficient of permeability corresponding to a voids ratio of 0.90 is 4.6×10^{-8} centimeters per second.

$$\text{Then } c = \frac{3,680 - 1,300}{1.00 - 0.80} \times \frac{9.2 \times 10^{-8}}{2 + 1.00 + 0.80} \times 60 = 0.0173$$

centimeter per minute.

Similar computations can be made, taking different values for e_1 and e_2 and the corresponding pressures, p_1 and p_2 from the curve, finding the coefficient of permeability corresponding to the average voids ratio as illustrated above, and substituting these values in equation (8).⁵ By this procedure, the values of c given in table 1 are computed. These values are plotted against the corresponding voids ratios in figure 3.

⁵ The pressures for voids ratios of 0.6, 1.2, and 1.4 were computed from the relation, $e = 1.050 - 0.442 \log p$.

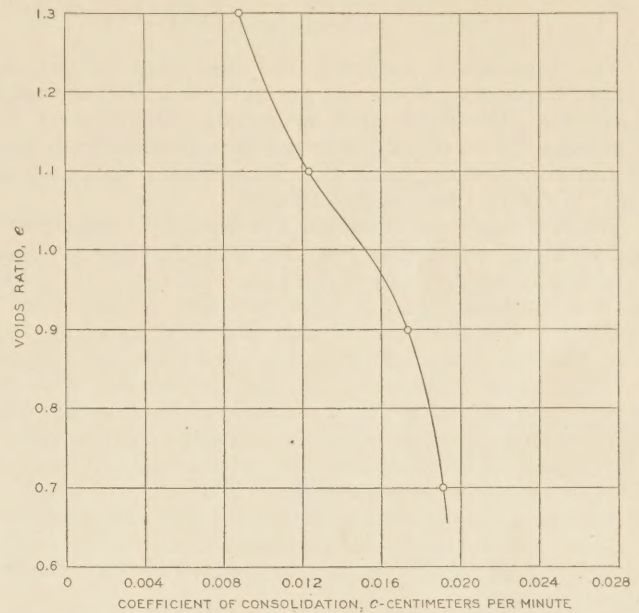


FIGURE 3.—RELATIONS BETWEEN COEFFICIENT OF CONSOLIDATION AND VOIDS RATIO.

TABLE 1.—Computed values of the coefficient of consolidation, c

Voids ratios		Pressures corresponding to voids ratios		Coefficient of permeability, k , corresponding to $\frac{e_1 + e_2}{2}$	Coefficient of consolidation, c
e_1	e_2	p_1	p_2		
		Grams per square centimeter	Grams per square centimeter	Centimeters per second	Centimeters per minute
1.40	1.20	161	458	22.6×10^{-8}	0.0088
1.20	1.00	458	1,300	10.2×10^{-8}	.0123
1.00	.80	1,300	3,680	4.6×10^{-8}	.0173
.80	.60	3,680	10,400	1.6×10^{-8}	.0190

This paper introduces only the most essential steps in some of the mathematical derivations used both for the computation of earth pressures and for computation of formulas applicable to the consolidation of relatively impervious, saturated strata caused by these pressures. For readers more concerned with direct application of the fundamental principles and formulas than with their development, a discussion of the use of the theory is presented in detail in part 1 of the paper. The mathematical derivations are included as part 2 for those interested also in the theoretical development.

PART 1.—APPLICATION OF THEORY OF SOIL CONSOLIDATION

The manner of applying the test data in practice varies, depending upon the pressure distribution conditions beneath the loaded area, the thickness of the compressible layer, and whether it is free to drain both at top and bottom, as in the compression test, or from only one of the bounding surfaces.

When a load is applied on the top of a compressible soil stratum, conditions may be such that the pressure resulting from the load is the same from top to bottom of the stratum, that is, the pressure distribution is uniform. On the other hand, the conditions may be such that the pressure varies, i. e., increases or decreases as the depth increases. In this paper the pressure change per unit of depth is considered to be either zero or constant. If it is zero, the pressure distribution is uniform; if it is constant, the pressure distribution is either triangular or trapezoidal. The cases of pressure distribution are illustrated in figure 4 and are as follows:

Case 1.—Rectangular pressure distribution. The pressure is uniform from top to bottom of the compressible stratum.

Case 2.—Triangular pressure distribution. The pressure varies from zero at the upper bounding surface of the compressible stratum to a finite value greater than zero at the lower bounding surface.

Case 3.—Triangular pressure distribution. The pressure varies from a finite value greater than zero at the top of the compressible layer to a pressure of zero at the bottom.

Case 4.—Trapezoidal pressure distribution. The pressure increases with increasing depth in the compressible stratum. The pressures at both top and bottom surfaces are finite values, each greater than zero.

Case 5.—Trapezoidal pressure distribution. The pressure diminishes with increasing depth in the compressible stratum. The pressures at both top and bottom surfaces are finite values, each greater than zero.

FORMULAS DEVELOPED FOR VARIOUS TYPES OF LOADED AREAS

The pressure distribution throughout the compressible layer depends on the shape and magnitude of the loaded area at the ground surface and on the load distribution over this area. In the examples cited in this paper the load distribution is taken as uniform over the loaded area.

The five types of loaded areas considered are:

- A.—A loaded area of great length and width.
- B.—A spot load.
- C.—A relatively short and narrow, rectangular loaded area.
- D.—A relatively narrow loaded area of great length.
- E.—A circular loaded area.

The pressure per unit area at any given depth below a loaded area of great extent is taken as equal to the unit pressure on the loaded surface plus the weight of superimposed earth above the point in question. No special formula is required for computations in this simple case.

To compute the pressure below a spot load, use is made of the equation,

$$p_z = \frac{KP}{z^2} \text{----- (9)}$$

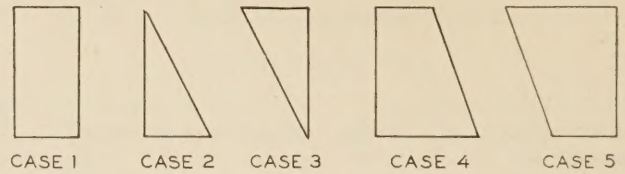


FIGURE 4.—FIVE CASES OF PRESSURE DISTRIBUTION IN SOIL STRATA.

where

$$K = \frac{3}{2\pi \left(1 + \frac{r^2}{z^2}\right)^{5/2}} \text{----- (10)}$$

P = the spot load; and

p_z = the vertical pressure per unit area at a vertical distance, z , below a spot load on the surface and at a horizontal distance, r , from the load.

For any distance on the vertical axis below the load, r is zero and $K = \frac{3}{2\pi}$.

Equations (9) and (10) can be used to determine the vertical pressure produced by any load, uniformly distributed over a rectangular area, provided the ratio of the longer side of the rectangle to z , the vertical depth to the point at which p_z is to be computed, is not greater than one-half. This condition is illustrated in figure 5. If, however, this ratio exceeds one-half, then the procedure is to subdivide the rectangle into rectangular elements (all of equal area) such that the ratio of the longer side of a single element to the vertical distance to the point in question is not greater than one-half. If the point at which it is desired to know the vertical pressure is at a very considerable depth below the loaded footing, then the procedure of subdivision may be unnecessary even in the case of large loaded areas. For example, the base of the Washington Monument is 125 feet square. At any point 250 feet beneath the base the ratio is $\frac{125}{250} = \frac{1}{2}$ and hence the entire loaded area may be considered as a point load at this depth.

When subdivision is necessary it is considered that the load on each element is concentrated at its center. To determine the unit pressure on the vertical center axis, the horizontal distance r from the center of each element to the center of the footing is computed and the ratio, $\frac{r}{z}$, is evaluated. This is substituted in the expression for K , equation (10), and K is computed for the element of area. Having done this with all of the different elements of area, the K values are added together and the sum is substituted in equation (9), the value for P in this equation being the total load on each individual subdivision of area. This procedure will be further illustrated later.

For a relatively narrow loaded area of great length, take a vertical cross section of the loaded strip. At a point on the vertical center axis, p_z is computed from the equation,

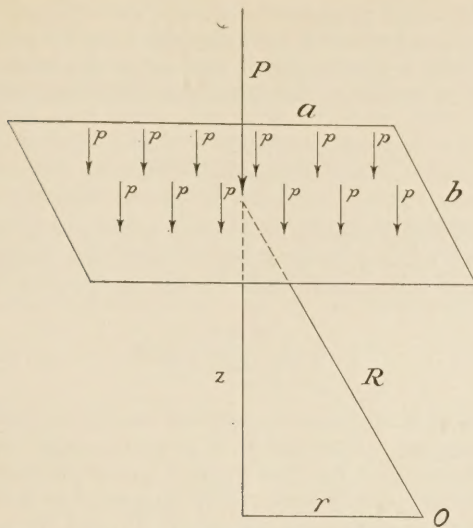


FIGURE 5.—RECTANGULAR AREA LOADED UNIFORMLY. (AREA OF TYPE C, ILLUSTRATING THE PRINCIPLES OF BOTH TYPES B AND C.)

$$p_z = \frac{p}{\pi}(\alpha + \sin \alpha) \dots\dots\dots (11)$$

where α = twice the angle, in radians, whose tangent is half the width of the strip divided by the vertical distance z to the point in question (see fig. 6); and p = the load per unit area of the footing.

To compute the maximum vertical pressure, p_z , at any point below a circular footing and on the center line (see fig. 6), use the equation,

$$p_z = p(1 - \cos^3 \beta) \dots\dots\dots (12)$$

where p = the load per unit area of the footing and β = the angle whose tangent is $\frac{r}{z}$ (see fig. 6).

RATE OF SETTLEMENT COMPUTED BY SIMPLE FORMULAS

If the compressible layer is sandwiched between two permeable sand-gravel layers, one above and one below, water can escape in two directions. If the compressible layer is overlaid with sand and rests on rock, water can escape from but one face of the stratum. There is a double filter in the first case and a single filter in the other. In either case, under a given pressure the thickness of the compressible layer is reduced to the same extent, ultimately. However, the rates of reduction in thickness of the compressible layer will obviously be different in the two cases. Consequently, the settlement during a relatively short time, say in 6 months or a year, will be greater if filtration can occur at both the upper and lower boundaries of the compressible stratum. The filtration effects a diminution in thickness and the earth's surface above the compressible stratum settles.

The total amount of vertical consolidation Q is computed by substitution of the data shown by the load-compression curve in the following expressions:

For soil sample,

$$Q = \left[\frac{e_1 - e_2}{1 + e_1} \right] d_1 \dots\dots\dots (13)$$

for soil stratum,

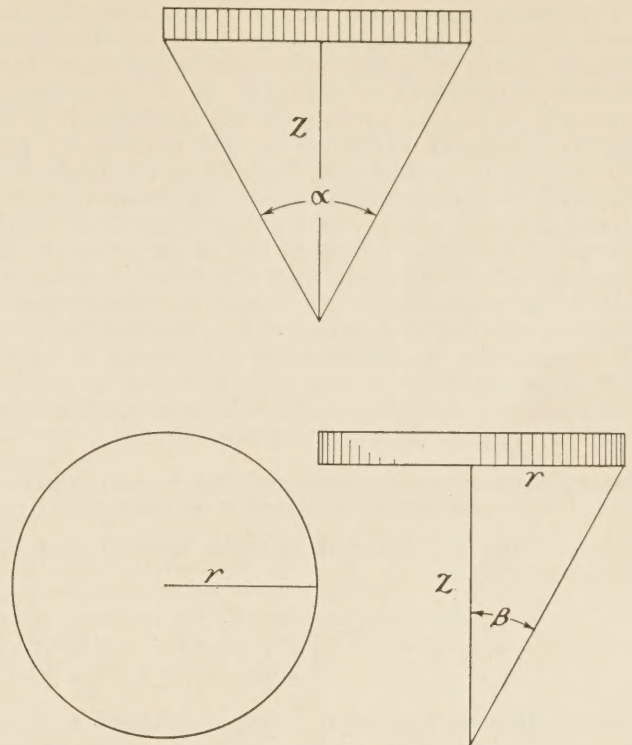


FIGURE 6.—TYPES OF LOADED AREAS. UPPER, VERTICAL CROSS-SECTION OF A STRIP OF GREAT LENGTH (LOADED AREA OF TYPE D). LOWER, A CIRCULAR LOADED AREA (TYPE E).

$$Q = \left[\frac{e_1 - e_2}{1 + e_1} \right] D_1 \dots\dots\dots (14)$$

where e_1 = voids ratio of the sample, taken as the average voids ratio in the compressible stratum prior to the application of additional load at the surface;

e_2 = voids ratio of the sample taken as the average voids ratio in the compressible stratum upon reaching equilibrium after the application of additional load at the surface;

d_1 = thickness of soil sample at apparent equilibrium at voids ratio, e_1 ; and

D_1 = the thickness of the soil stratum at voids ratio, e_1 .

The rates at which soil strata settle are computed from test data and field observations by simple formulas.

The theoretical time-settlement relation for soil samples or strata sandwiched between two porous layers and with either rectangular, triangular, or trapezoidal pressure distribution or for strata with rectangular pressure distribution and with a porous layer only at the upper face is given by the expression,

$$q_1 = 1 - \frac{8}{\pi^2} \left(e^{-N} + \frac{1}{9} e^{-9N} + \frac{1}{25} e^{-25N} + \dots \right) \dots\dots (15)$$

where q_1 = percentage of ultimate consolidation at the end of a definite period of time, t , of load application;

e = Napierian base.

The exponent, N , is related to the thickness of the soil sample, the period of load application, t , and the speed of egress of water from the soil sample or stratum.

Experience has shown that the time-consolidation curves furnished by the compression test on compressible soils (see fig. 2, lower curve) conform in shape with the theoretical curve of equation (15) from q_1 equals zero to q_1 equals 90 percent on the theoretical curve. The experimental values of q_1 attributable to permeability alone, are usually from 60 to 100 percent of the actual ultimate consolidation as obtained in the laboratory with the given loading conditions. Only laboratory values of q_1 within the range in which the experimental curve agrees with the curve showing the theoretical relationship are valid for computations involving the use of equation (15). Beyond the limit of consolidation as effected by the egress of water, further consolidation of most soils is caused by particle rearrangement and does not result from permeability as assumed in the derivation of equation (15).

TIME-CONSOLIDATION RELATIONS GIVEN FOR UNIFORM, TRIANGULAR, AND TRAPEZOIDAL PRESSURE DISTRIBUTIONS

Considering a laboratory sample, the equation for the exponent N is

$$N = \frac{\pi^2 ct}{4 \left(\frac{d_0}{2}\right)^2}$$

When a definite numerical value is assigned to q_1 , N has a numerical value, designated as N_1 . The expression then becomes

$$t = \frac{N_1 \left(\frac{d_0}{2}\right)^2}{2.47c} \text{----- (16)}$$

- where t = period of load application, in minutes.
- d_0 = thickness in centimeters that the laboratory sample would have if it were possible for the voids ratio to equal zero.
- d_0 = twice the maximum distance water travels when flowing from a sample of d_0 thickness sandwiched between two filters.
- c = coefficient of consolidation of compressible soil, in centimeters per minute.

To express t of equation (16) in years instead of minutes, and d_0 in feet instead of centimeters, the following computations are necessary:

$$1 \text{ centimeter} = \frac{1}{2.54 \times 12} \text{ feet}$$

and

$$1 \text{ minute} = \frac{1}{365.24 \times 24 \times 60} \text{ years.}$$

Then

$$t = \frac{N_1 \left(\frac{d_0}{2}\right)^2}{2.47c} \times \frac{(2.54 \times 12)^2}{365.24 \times 24 \times 60} = \frac{N_1 \left(\frac{d_0}{2}\right)^2}{1,400c} \text{ years.}$$

In this equation d_0 is in feet.

For a compressible stratum in the field, sandwiched between two permeable layers of soil, the time required for a given percentage of consolidation to take place is computed from the equation,

$$t = \frac{N_1 \left(\frac{D_0}{2}\right)^2}{1,400c} \text{ years----- (17)}$$

where D_0 = the thickness in feet of the stratum at $e=0$.

When a compressible soil stratum rests on rock and is covered with a porous layer, and when the pressure distribution is uniform, the time-consolidation relation is

$$t = \frac{N_1 D_0^2}{1,400c} \text{ years----- (18)}$$

When a soil stratum rests on impermeable rock and is covered with a porous layer, and the pressure distribution is triangular (case 2), the expression becomes

$$t = \frac{N_2 D_0^2}{1,400c} \text{ years----- (19)}$$

where N_2 is a definite numerical value corresponding to an assigned value for q_2 , the percentage consolidation at the time, t , for this type of pressure distribution and boundary conditions. For values of q_1 and q_2 , the corresponding numerical values of N_1 and N_2 are given in table 2.

For triangular pressure distribution (case 3) with a permeable layer only at the upper bounding surface of the compressible stratum, the time-settlement relation is

$$t = \frac{N_3 D_0^2}{1,400c} \text{ years----- (20)}$$

For values of q_3 the corresponding numerical values of N_3 are given in table 2. Here q_3 denotes the percentage of consolidation at any time, t , with this type of pressure distribution (case 3) and with the stated boundary conditions.

The procedures followed in computing the theoretical values of N_1 , N_2 , and N_3 , corresponding to values of q_1 , q_2 , and q_3 will be shown on pages 7 and 8 preceding the illustrative examples.

TABLE 2.—Values of N_1 , N_2 , and N_3 corresponding to values of q_1 , q_2 , and q_3 for cases 1, 2, and 3, respectively

Consolidation $q_1, q_2,$ or q_3	Numerical values of—		
	N_1	N_2	N_3
Percent			
0	0.00	0.00	0.00
5	.005	.06	.002
10	.02	.12	.005
15	.04	.18	.01
20	.08	.25	.02
25	.12	.31	.03
30	.17	.39	.06
35	.24	.47	.09
40	.31	.55	.12
45	.39	.63	.17
50	.49	.73	.24
55	.59	.84	.32
60	.71	.95	.42
65	.84	1.08	.54
70	1.00	1.24	.69
75	1.18	1.42	.88
80	1.40	1.64	1.08
85	1.69	1.93	1.36
90	2.09	2.35	1.77
95	2.80	3.02	2.54
100	Infinity	Infinity	Infinity

With trapezoidal pressure distribution, the pressure resulting from the applied load may either increase (case 4) or decrease (case 5) with depth, and has a finite value at both the upper and lower boundaries or surfaces of the compressible stratum. The ratio of the pressure at the upper boundary to that at the lower is designated as u . The value of u is unity for uniform and either zero or infinity for triangular pressure dis-

tribution. For trapezoidal pressure distribution, (case 4), u is less than 1 and greater than zero. An interpolation factor, J , is applied in this case in computing the rate of settlement if the compressible stratum is overlaid with sand and rests on rock. When this condition obtains, interpolation is between the two extremes of uniform and triangular pressure distributions. With the compressible stratum bounded below by sand and above by rock, and with a trapezoidal pressure distribution such that u exceeds unity, (case 5) an interpolation factor, J' , is used in computing the rate of consolidation.

The expressions for J and J' in terms of u are as follows:

$$\text{For case 4, } J = \frac{\log \left[\frac{2(\pi-2)u+4}{\pi(1+u)} \right]}{\log \frac{4}{\pi}} \quad \text{----- (21)}$$

$$\text{For case 5, } J' = \frac{\log \left[\frac{u(\pi-2)+2}{(\pi-2)(u+1)} \right]}{\log \left[\frac{\pi}{2(\pi-2)} \right]} \quad \text{----- (22)}$$

Values for J and J' corresponding to values assigned to u are listed in table 3.

The time-settlement relations for cases 4 and 5, trapezoidal pressure distributions, are:

$$\text{Case 4, } t = \frac{N_4 D_0^2}{1,400 c} \text{ years ----- (23)}$$

$$\text{Case 5, } t = \frac{N_5 D_0^2}{1,400 c} \text{ years ----- (24)}$$

$$\text{where } N_4 = N_1 + J(N_2 - N_1) \quad \text{----- (25)}$$

$$\text{and } N_5 = N_3 + J'(N_1 - N_3) \quad \text{----- (26)}$$

TABLE 3.—Values of u , J and J' used in computing the rates of settlement for the two types of trapezoidal pressure distribution

Pressure increasing with depth, u varying from 0 to 1		Pressure diminishing with depth, u varying from 1 to infinity	
Value of u	Value of J	Value of u	Value of J'
0.0	1.00	1.0	1.00
.1	.84	1.2	.92
.2	.69	1.4	.86
.3	.57	1.6	.80
.4	.46	1.8	.75
.5	.36	1.9	.73
.6	.27	2.0	.71
.7	.19	2.2	.67
.8	.12	2.5	.62
.9	.06	3.0	.55
1.0	.00	3.5	.50
		4.0	.45
		4.5	.42
		5.0	.39
		6.0	.34
		7.0	.30
		8.0	.27
		9.0	.25
		10.0	.23
		12.0	.20
		15.0	.17
		20.0	.13
		100.0	.02

FAIRLY CLOSE AGREEMENT FOUND BETWEEN THEORETICAL AND TEST VALUES

The time-consolidation curve (fig. 2) represents the average curve for pressures of $p=1.079, 2.118, 4.196,$ and 8.352 kilograms per square centimeter. The time-consolidation curve for the load $p=0.61$ kilogram per square centimeter was obviously influenced by experi-

mental error and therefore was not included in computing the values for the average curve.

The divergence of the average curve from the curve given by equation (15) may be determined by means of equation (16) as follows:

For different values of q_1 , say 25, 50, and 75 percent, the values of t are obtained from the curves. The average values of e at the different stages of loading are also obtained from these curves. Then the values of c , corresponding to the average values of e , are read from the curve in figure 3; the value of d_0 is computed; the numerical values are substituted in equation (16); and the values for N_1 may thus be computed, assuming that the relationship in equation (16) obtains.

The thickness of the sample, d , at a voids ratio, $e=1.670$, was found in the test to be 1.130 centimeters. Therefore the thickness d_0 when $e=0$ would be 0.423 centimeter.

The values for N_1 thus obtained (see table 4) for equal values of q_1 are averaged and found to be 0.11 for $q_1=25, 0.44$ for $q_1=50,$ and 1.30 for $q_1=75.$

The theoretical values of N_1 for the same values of q_1 are shown in the last column of table 4. The agreement between the determined and theoretical values is fairly good. The procedure in computing these theoretical values of N_1 follows:

TABLE 4.—Values for $q_1, e, c,$ and N_1 obtained from laboratory data

Pressure interval	Consolidation, q_1	Average voids ratio, e	Values of c corresponding to average e	Values of N_1 computed from data ¹	Values of N_1 computed from theory
Kg per sq. cm 0.610-1.079....	Percent 25	1.17	0.0108	0.10	0.12
	50	1.13	.0116	.49	.49
	75	1.08	.0140	1.47	1.18
1.079-2.118....	25	1.60	.0152	.13	.12
	50	.97	.0159	.42	.49
	75	.94	.0166	1.20	1.18
2.118-4.196...	25	.87	.0177	.12	.12
	50	.84	.0180	.42	.49
	75	.80	.0183	1.32	1.18
4.196-8.352...	25	.74	.0188	.09	.12
	50	.72	.0189	.44	.49
	75	.69	.0191	1.22	1.18

¹ For $q_1=25$ percent, average N_1 (computed from data in column 5)=0.11. For $q_1=50$ percent, average N_1 (computed from data in column 5)=0.44. For $q_1=75$ percent, average N_1 (computed from data in column 5)=1.30.

COMPUTATIONS GIVEN FOR VARIOUS TYPES OF PRESSURE DISTRIBUTION AND BOUNDARY CONDITIONS

Case 1.—The computations for case 1 cover uniform pressure distribution, irrespective of boundary conditions, and triangular and trapezoidal pressure distribution when the compressible stratum is bounded on both top and bottom by permeable strata. To compute the value of N_1 corresponding to any definite value assigned to q_1 in the expression,

$$q_1 = 1 - \frac{8}{\pi^2} \left(e^{-N} + \frac{1}{9} e^{-9N} + \frac{1}{25} e^{-25N} + \dots \right) \quad \text{----- (15)}$$

one may, as an approximation, neglect all terms of e except the first. Then

$$q_1 = 1 - \frac{8}{\pi^2} e^{-N} \quad \text{----- (27)}$$

OR

$$1 - q_1 = \frac{8}{\pi^2} e^{-N} \quad \text{----- (28)}$$

For $q_1 = \frac{1}{2}$, equation (28) becomes

$$\frac{1}{2} = \frac{8}{\pi^2} e^{-N_1}$$

and

$$\log_e \frac{1}{2} = \log_e \frac{8}{\pi^2} - N_1,$$

where "log_e" designates the natural logarithm.

$$N_1 = \log_e \frac{8}{\pi^2} - \log_e \frac{1}{2} = 2.3026 \left(\log_e \frac{8}{\pi^2} - \log_e \frac{1}{2} \right),$$

where "log" denotes the base 10 system of logarithms. Hence $N_1 = 0.48$.

The more accurate procedure would be to assign values to N , include several terms of the series, and solve for q_1 . In this manner it is found that for $N_1 = 0.49$ the value of q_1 is 0.50 or 50 percent. (See table 2.) However, it is seen that by neglecting all terms of e except the first in equation (15), the error, $0.49 - 0.48 = 0.01$, is small.

Case 2.—Triangular pressure distribution with zero pressure at the upper boundary, when the compressible stratum is bounded by an impermeable stratum below and a permeable stratum above. For this condition the theoretical time-settlement relationship is given by the expression,

$$q_2 = 1 - \frac{32}{\pi^3} \left(e^{-N} - \frac{1}{27} e^{-9N} + \frac{1}{125} e^{-25N} - \dots \right) \dots (29)$$

By neglecting all terms of e except the first, one may write,

$$1 - q_2 = \frac{32}{\pi^3} e^{-N} \dots (30)$$

for $q_2 = \frac{1}{2}$,
$$N_2 = \log_e \frac{32}{\pi^3} - \log_e \frac{1}{2} = 0.72.$$

This compares favorably with the value, $N_2 = 0.73$ for $q_2 = 50$ percent, table 2, obtained by taking more terms in the series than the first in equation (29).

Case 3.—Triangular pressure distribution with zero pressure at the lower boundary, when the compressible stratum is bounded by an impermeable stratum below and a permeable stratum above. It will be shown in part 2 of this paper that

$$q_3 = 2q_1 - q_2 \dots (31)$$

The simplest procedure in computing values of N_3 that correspond to values of q_3 is to take the case, $N_1 = N_2 = N_3$. For $N_1 = N_2 = N_3 = 0.12$, it is seen in table 2 that $q_1 = 0.25$ and $q_2 = 0.10$, so that $q_3 = 2 \times 0.25 - 0.10 = 0.40$. That is for $q_3 = 40$ percent, $N_3 = 0.12$.

Case 4.—Trapezoidal pressure distribution with pressure increasing with depth, when the compressible stratum is bounded by an impermeable stratum below and a permeable stratum above. The value, N_4 , is computed from equation (25), which is $N_4 = N_1 + J(N_2 - N_1)$, applicable for $q_4 = q_1 = q_2$. J in turn is computed from equation (21) for any given value of u . For example, when $u = 0.50$, $J = 0.36$ (see table 3). For $q_1 = q_2 = q_4 = 50$ percent, $N_1 = 0.49$ and $N_2 = 0.73$. Then

$$N_4 = 0.49 + 0.36(0.73 - 0.49) = 0.58.$$

When this value is substituted for N_4 in equation (23),

$$t = \frac{N_4 D_0^2}{1,400 c} \text{ years,}$$

the time required to produce 50 percent of the ultimate consolidation is easily calculated. The value of c is obtained from laboratory data according to the procedure already indicated and $D_0 = \frac{D_1}{1 + e_1}$ as already shown (equation 2). The thickness, D_1 , of the compressible stratum prior to applying a load at the surface is known from the test borings. If $D_1 = 20$ feet and $e_1 = 1.5$, then $D_0 = \frac{20}{2.5} = 8$ feet. And if $c = 0.01$ and $N_4 = 0.58$, then $t = \frac{0.58 \times 64}{14} = 2.65$ years. This is the time required for 50 percent of the ultimate consolidation to take place under the given conditions, since a value of $N_4 = 0.58$ corresponds to 50 percent consolidation.

Case 5.—Trapezoidal pressure distribution with pressure decreasing with depth, when the compressible stratum is bounded by an impermeable stratum below and a permeable stratum above. For $q_1 = q_3 = q_5$, the value, N_5 is computed from equation (26), which is,

$$N_5 = N_3 + J'(N_1 - N_3),$$

for any given value of u . From table 3, $J' = 0.71$ when $u = 2$. For $q_1 = q_3 = q_5 = 50$ percent, $N_1 = 0.49$ and $N_3 = 0.24$ from table 2. Then by substitution, $N_5 = 0.24 + 0.71(0.49 - 0.24) = 0.42$. Now take the conditions already assumed for case 4, that is, $c = 0.01$, $e_1 = 1.5$, the average voids ratio in the stratum prior to loading, and $D = 20$ feet. Then

$$t = \frac{N_5 D_0^2}{1,400 c} \text{ years} = \frac{0.42 \times 64}{14} \text{ or } 1.9 \text{ years.}$$

DISCUSSION OF CONDITIONS GOVERNING ILLUSTRATIVE EXAMPLES THAT FOLLOW

In the following examples it is assumed that the material composing the compressible stratum has the consolidation characteristics derived from computations of the data plotted in figures 2 and 3, and that the weight per cubic foot of this material as well as that of the sand above it is 100 pounds. Cases 1 to 5 inclusive are linear types of pressure distribution, that is, the pressure distribution diagrams are composed of straight lines. All of these five types of pressure distribution give the same time-consolidation relation, equation (17), when the compressible stratum is bounded both above and below by drainage courses (boundary condition A). Equation (18) is the time-consolidation relation when there is but one drainage course (boundary condition B) and the pressure distribution is rectangular (case 1). When there is but one drainage course, the stratum resting on rock and being overlaid with sand, equations (19), (20), (23), and (24) are the time-consolidation relations for cases 2, 3, 4, and 5, respectively, and computations are made with the aid of tables 2 and 3 and the interpolation formulas, making use of J and J' , that is, equations (21), (22), (25), and (26).

Cases 1 to 5 inclusive may be defined in terms of u , the ratio of the pressure at the upper surface of the compressible stratum to the pressure at the lower surface, that is:

- Case 1, rectangular pressure distribution, $u=1$.
- Case 2, triangular pressure distribution, $u=0$.
- Case 3, triangular pressure distribution, $u=\infty$.
- Case 4, trapezoidal pressure distribution, u varies from 0 to 1.
- Case 5, trapezoidal pressure distribution, u varies from 1 to infinity.

In solving any problem involving case 4, the formulas and principles of cases 1 and 2 are applied. When the pressure distribution is of the type case 5, application is made of the formulas and principles that pertain to cases 1 and 3. Cases 1 and 2 are both limiting conditions of case 4, and cases 1 and 3 are both limiting conditions of case 5. The types of pressure distribution most frequently encountered in foundation problems are cases 1 and 5, more especially 5.

Example 1.—Case 1 pressure distribution; type A loading. Consider a uniform load of 3 tons per square foot over an area of great extent. Immediately beneath the loaded area there is a layer of sand, 10 feet thick. Below the sand is a compressible stratum also 10 feet thick and below this stratum there may be either (A) sand or (B) rock. It is desired to know the total settlement and the time required for 25, 50, and 75 percent of the total settlement to take place.

Use equation (17) for boundary condition A and equation (18) for boundary condition B for the time-consolidation part of the problem. It is assumed that consolidation of the compressible stratum resulting from its own weight and that of the sand above it has reached completion prior to application of the surface load.

Prior to loading, the pressure is 1,000 pounds per square foot at the top and 2,000 pounds per square foot at the bottom of the compressible layer. In figure 2, these pressures correspond to voids ratios of 1.28 and 1.06 respectively. A pressure of 1 kilogram per square centimeter is very nearly 1 ton per square foot so that the units of pressure of figure 2 (load-compression curve) are directly applicable. The average voids ratio prior to loading is then

$$\frac{1.28 + 1.06}{2} = 1.17 = e_1.$$

After loading, the pressure at the top of the stratum is 3.5 tons and at the bottom, 4 tons per square foot. These pressures correspond to the voids ratios (see fig. 2), 0.80 and 0.78, respectively, the average, e_2 being 0.79. The total settlement is computed by equation (14), that is,

$$Q = \frac{e_1 - e_2}{1 + e_1} \times D_1 = \frac{1.17 - 0.79}{1 + 1.17} \times 10 = 1.75 \text{ feet.}$$

If there were no voids whatsoever in the material composing the compressible stratum, its thickness would be

$$D_0 = \frac{1}{1 + e_1} D_1 = \frac{1}{2.17} \times 10 = 4.61 \text{ feet.}$$

The average voids ratio is reduced by the surface loading from $e_1=1.17$ to $e_2=0.79$. Within this interval there is an average e of $\frac{1.17 + 0.79}{2} = 0.98$. From figure 3, a voids ratio of 0.98 corresponds to a value for c , the coefficient of consolidation, equal to 0.0157.

Now apply equation (17) for boundary condition A, the compressible stratum sandwiched between two drainage courses. The times required for 25, 50, and 75 percent of ultimate consolidation (1.75 feet) to take place are, using table 1 for values of N_1 ,

$$\text{for } q_1=25 \text{ percent, and } N_1=0.12, t = \frac{N_1 \left(\frac{D_0}{2}\right)^2}{1,400 c}$$

$$= \frac{0.12 \times \left(\frac{4.61}{2}\right)^2}{1,400 \times 0.0157} = 0.03 \text{ year or 11 days,}$$

$$\text{for } q_1=50 \text{ percent, } N_1=0.49, t = \frac{0.49 \times \left(\frac{4.61}{2}\right)^2}{1,400 \times 0.0157} = 0.12 \text{ year} = 44 \text{ days,}$$

$$\text{for } q_1=75 \text{ percent, } N_1=1.18, t = \frac{1.18 \times \left(\frac{4.61}{2}\right)^2}{1,400 \times 0.0157} = 0.285 \text{ year} = 104 \text{ days.}$$

For condition B, the stratum covered with sand and resting on rock, the values of N_1 corresponding to values of q_1 are the same as for condition A. Solving equation (18) for $q_1=25$ percent, $t = \frac{0.12 \times (4.61)^2}{1,400 \times 0.0157} = 0.12$ year = 44 days. Similarly, for $q_1=50$ percent, $t = 0.48$ year, and for $q_1=75$ percent, $t = 1.14$ years.

Twenty-five percent of 1.75 feet, the ultimate settlement, is 5.2 inches. For condition A, this amount of settlement occurs in 11 days; for condition B, 44 days are required for this amount of settlement.

Example 2.—Case 5 pressure distribution; type C loading. Instead of a loading area of great extent as in example 1, there is a rectangular rigid footing, 10 feet wide and 30 feet long, carrying a uniform load of 3 tons per square foot. This is at the ground surface above the same compressible stratum described in example 1. The average voids ratio, e_1 , of the compressible layer prior to loading is taken as 1.17; its thickness prior to loading is 10 feet; and the weight of the sand at the upper level of the stratum is 1,000 pounds per square foot.

The vertical distance, z , of the footing above the upper boundary of the stratum is 10 feet. The ratio of the length of the footing to z is 3. This ratio is too large to warrant taking the uniform load on the footing as a point load at its center. For purposes of computations it is necessary to consider the footing as subdivided into rectangles such that the ratio of the length of a single rectangle to z does not exceed one-half (see figs. 5 and 7). The footing may be divided into twelve 5-foot squares to meet this requirement.

The total load on the footing is $3 \times 10 \times 30 = 900$ tons. The load on a single square is 75 tons and it is considered that this load is concentrated at the center of the square. The horizontal distance from the center of a square to the center of the 10-by-30-foot foundation is r and this is found from the properties of a right triangle. For example, the center of square number 5 from C, the center of the footing, is the square root of $\left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2$, or $\frac{5}{\sqrt{2}} = r$. The same value for r is obtained

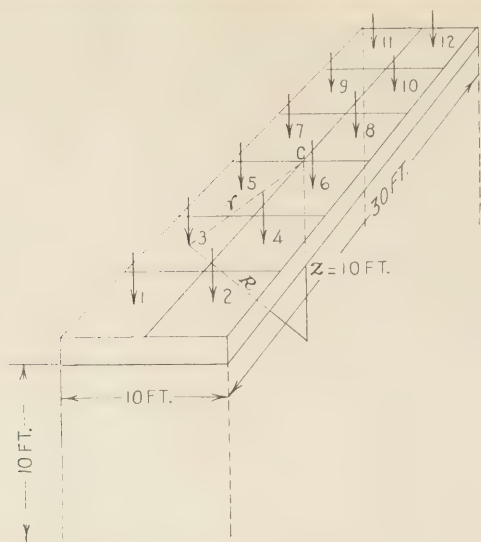


FIGURE 7.—RECTANGULAR LOADED AREA (TYPE C).

for squares 6, 7, and 8. The ratio, $\frac{r}{z}$, appearing in equation (10) is $\frac{5}{10\sqrt{2}}$ or $\frac{1}{2\sqrt{2}}$ for squares 5, 6, 7, and 8. These values and those obtained for the other squares (fig. 7) are given in table 5, together with the value of K obtained by substituting the value for $\frac{r}{z}$ in equation (10),

$$K = \frac{3}{2\pi \left(1 + \frac{r^2}{z^2}\right)^{5/2}}$$

The three values for K , corresponding to the three groups of load numbers given in the first column of table 5, are added together, their sum being 0.5404. This must be multiplied by 4 to give K for the entire footing. The vertical stress at a point 10 feet below C and on the vertical center axis through C is then calculated from equation (9). That is,

$$p_z = \frac{KP}{z^2} = \frac{75}{100} \times 4 \times 0.5404 = 1.62 \text{ tons per square foot.}$$

TABLE 5.—Computations of vertical stresses below a rectangular footing 10 by 30 feet

Load numbers (see fig. 7)	Distance, r	Ratio, $\frac{r}{z}$	Coefficient K
5, 6, 7, and 8	$\frac{5}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}} = 0.354$	0.3555
3, 4, 9, and 10	$\frac{5\sqrt{10}}{2}$	$\frac{\sqrt{10}}{4} = 0.790$.1420
1, 2, 11, and 12	$\frac{5\sqrt{26}}{2}$	$\frac{\sqrt{26}}{4} = 1.274$.0429
Total			.5404

By taking $z=20$ feet and computing in the same manner, it is found that at the lower boundary of the compressible stratum and on the vertical center axis of the footing, p_z is 0.73 ton per square foot. The pressure is a maximum for points along the vertical center axis of the footing and the distribution of this maximum pressure is trapezoidal, case 5. At points removed from the vertical center axis there is also case 5 pres-

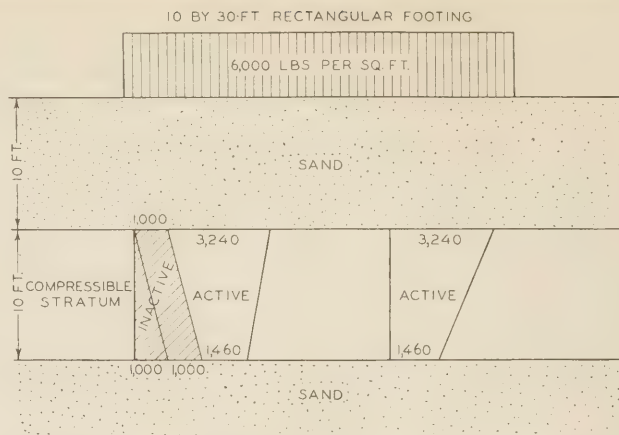


FIGURE 8.—DISTRIBUTION OF MAXIMUM PRESSURE UNDER RECTANGULAR FOOTING. SOIL LAYER CONTAINED BETWEEN TWO PERMEABLE SAND LAYERS.

sure distribution, but the value of p_z at any level is less than the value of p_z for the same level on the vertical center axis. For computations consider only the maximum pressures, which are along the vertical center axis.

The shaded trapezoidal area marked "inactive" in figure 8, denotes the pressure distribution resulting from the weight of earth material alone. The trapezoidal figure marked "active" denotes the distribution of maximum pressure in the compressible stratum that results from the uniform load on the footing. It is again assumed that consolidation of the stratum caused by its own weight and that of the sand is complete at the time the surface load is applied and hence all of this pressure has been transferred from water to soil and therefore cannot act in combination with the applied surface load in causing consolidation. The applied load producing the active trapezoidal pressure distribution produces further consolidation beyond that already effected by the weight of earth material.

When the surface load is applied, the pressure per square foot at the upper boundary of the compressible layer is $1,000 + 3,240 = 4,240$ pounds or 2.12 tons. That at the lower boundary is 3,460 or 1.73 tons. From figure 2, these pressures correspond to voids ratios of 0.90 and 0.94, respectively, the average value of e being $0.92 = e_2$. Prior to loading, the average value of e , as found in example 1, is 1.17. The ultimate settlement is therefore

$$Q = \frac{1.17 - 0.92}{1 + 1.17} \times 10 = 1.15 \text{ feet} = 13.8 \text{ inches.}$$

The average voids ratio is reduced by the surface loading from 1.17 to 0.92. The average of these is 1.045 and the corresponding value of c from figure 3 is 0.0138.

Example 2 (continued).—Boundary condition A; the stratum is sandwiched between sand layers. For the time-consolidation computations, again apply equation (17) for boundary condition A (see fig. 8).

$$\text{For } q_1 = 25 \text{ percent, } t = \frac{N_1 \left(\frac{D_0}{2}\right)^2}{1,400 c} = \frac{0.12 \times \left(\frac{4.61}{2}\right)^2}{1,400 \times 0.0138} = 0.033$$

year = 12 days.

Similarly, $t = 0.135$ year = 49 days for $q_1 = 50$ percent and 0.325 year = 119 days for $q_1 = 75$ percent.

These values are nearly equal to those computed for boundary condition A, example 1, and would be the

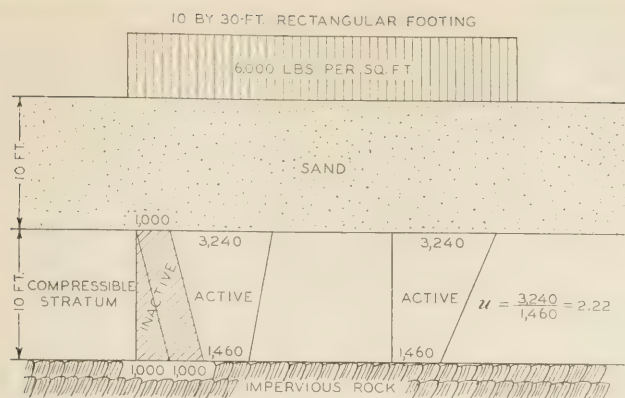


FIGURE 9.—DISTRIBUTION OF MAXIMUM PRESSURE UNDER RECTANGULAR FOOTING. SOIL LAYER OVERLAID BY SAND AND RESTING ON ROCK.

same were it not for small variations in c as the pressure is varied. For most practical purposes, when boundary condition A obtains, the time-consolidation relation can be considered independent of the amount of pressure and the type of pressure distribution.

Example 2 (continued).—Boundary condition B; the compressible stratum rests on rock and is overlaid with sand. When condition B obtains in this example (see fig. 9), use equations (24) and (26), together with tables 2 and 3. The total settlement, Q , is the same for both boundary conditions, A and B. The value for u (consider only the active pressure, fig. 9) is 2.22. Then J' is obtained from table 3 and is taken as 0.67. From table 2, when $q_1 = q_3 = q_5 = 25$ percent, $N_1 = 0.12$ and $N_3 = 0.03$, and from equation (26),

$$N_5 = N_3 + J'(N_1 - N_3) = 0.03 + 0.67(0.12 - 0.03) = 0.09.$$

This value for N_5 is substituted in equation (24) and taking $c = 0.0138$, then for $q_5 = 25$ percent,

$$t = \frac{N_5 D_0^2}{1,400 c} = \frac{0.09 \times (4.61)^2}{1,400 \times 0.0138} = 0.099 \text{ year} = 36 \text{ days.}$$

For $q_1 = q_3 = q_5 = 50$ percent, N_1 (table 2) = 0.49 and $N_3 = 0.24$.

Then $N_5 = 0.24 + 0.67(0.49 - 0.24) = 0.41$ and

$$t = \frac{0.41 \times (4.61)^2}{1,400 \times 0.0138} = 0.451 \text{ year} = 165 \text{ days.}$$

Similarly for $q_5 = 75$ percent,

$$t = \frac{1.08 \times (4.61)^2}{1,400 \times 0.0138} = 1.19 \text{ years} = 435 \text{ days.}$$

Example 3.—Case 4, pressure distribution; type E loading; boundary condition B. It would seem that case 4 pressure distribution is a comparatively rare occurrence. Problems of this type may be encountered, however, when consolidation produced by the weight of earth material alone is incomplete at the time of load application at the ground surface. Figure 10 illustrates such a condition. If it is assumed that consolidation in the compressible stratum 5 feet thick resulting from the weight of sand above is only 50 percent completed, then this weight exerts an active pressure which is zero at the top and 1,000 pounds per square foot at the bottom of the compressible layer. The distribution of active pressure resulting from the weight of sand alone is, therefore, triangular. It is assumed that consolidation of the

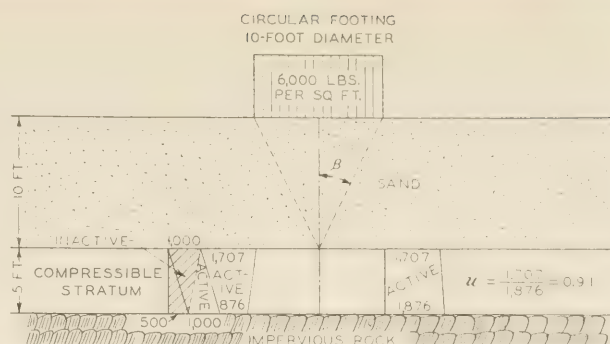


FIGURE 10.—DISTRIBUTION OF MAXIMUM PRESSURE UNDER CIRCULAR FOOTING. SOIL LAYER OVERLAID BY SAND AND RESTING ON ROCK.

compressible layer as produced by its own weight is complete at the time of loading.

Referring to figure 10, a load of 3 tons per square foot is applied over a rigid circular footing 10 feet in diameter. It is desired to determine the total settlement and the rate of settlement of the compressible layer which rests on rock and is covered with the sand layer 10 feet thick, 50 percent of the consolidation producible by the weight of sand being completed at the time of load application.

To compute the pressure, p_z , at a point on the vertical centerline 10 feet below the rigid footing, substitute values in equation (12). The cosine of angle β (fig. 10)

is $\frac{10}{\sqrt{125}} = \frac{2}{\sqrt{5}}$ and p equals 3 tons per square foot. Then

$$p_z = p(1 - \cos^3 \beta) = 3 \left(1 - \frac{8}{5\sqrt{5}} \right) = 0.854 \text{ ton per square foot.}$$

At a point 15 feet below the footing and on the vertical center axis, $p_z = 0.438$ ton per square foot. Therefore, the applied load produces a pressure of 1,707 pounds per square foot at the upper boundary and 876 pounds per square foot at the lower boundary of the compressible layer.

Prior to loading, e at the upper surface of the layer is 1.28, corresponding to a load of 1,000 pounds per square foot, and at the lower boundary it is 1.44, corresponding to a load of 500 pounds per square foot. The average is $1.36 = e_1$. The voids ratios (see fig. 2) corresponding to pressures of 1,000 + 1,707 pounds per square foot at the top and 500 + 1,876 pounds per square foot at the bottom of the layer are found (fig. 2) to be 0.99 and 1.01, respectively. The average is $1.00 = e_2$. The total settlement is, therefore,

$$Q = \left[\frac{1.36 - 1.00}{1 + 1.36} \right] \times 5 = 0.76 \text{ foot.}$$

It is important to note that prior to loading, soil at the top of the compressible stratum has reached equilibrium under the weight of sand (1,000 pounds per square foot) and that soil at the bottom of the stratum has not as yet been consolidated by the sand but has been consolidated by the weight of compressible soil, i. e., 500 pounds per square foot. Therefore, the voids ratio at the top of the layer prior to loading corresponds to a pressure of 1,000 and that at the bottom to a pressure of 500 pounds per square foot. After a very long period of time subsequent to loading, the soil at the top of the compressible layer will be in equilibrium with a pressure of 2,707 pounds per square foot and that at the bottom with a

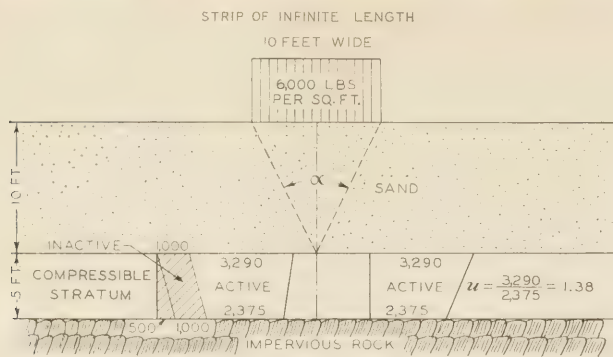


FIGURE 11.—DISTRIBUTION OF MAXIMUM PRESSURE UNDER STRIP OF INFINITE LENGTH. SOIL LAYER OVERLAID BY SAND AND RESTING ON ROCK.

pressure of 2,376 pounds per square foot. These pressures correspond to definite voids ratios, found experimentally (see fig. 2), and are used in computing Q .

When load is applied, the active pressure at the top of the layer is 1,707 and that at the bottom is 1,876 pounds per square foot. Hence $u = \frac{1,707}{1,876} = 0.91$, a value between 0 and 1 (case 4). Apply equations (23) and (25) and use the values for N_1 and N_2 corresponding to different values for $q_1 = q_2$, table 2. Values of J are found from table 4, and for $u = 0.91$, $J = 0.06$. For $q_1 = q_2 = 25$ percent, $N_1 = 0.12$, $N_2 = 0.31$, and

$$N_4 = N_1 + J(N_2 - N_1) = 0.12 + 0.06(0.31 - 0.12) = 0.13$$

$$D_0 = \frac{1}{1 + e_1} \times 5 = \frac{1}{2.36} \times 5 = 2.12 \text{ feet.}$$

$$\text{For } e = \frac{e_1 + e_2}{2} = 1.18, c = 0.0106 \text{ (fig. 3).}$$

$$\text{Then for } q_4 = 25 \text{ percent, } t = \frac{N_4 D_0^2}{1,400c} = \frac{0.13 \times (2.12)^2}{1,400 \times 0.0106} = 0.039 \text{ year} = 14 \text{ days.}$$

$$\text{Similarly, for } q_4 = 50 \text{ percent, } t = \frac{0.50 \times (2.12)^2}{1,400 \times 0.0106} = 0.15 \text{ year} = 55 \text{ days,}$$

$$\text{and for } q_4 = 75 \text{ percent, } t = \frac{1.19 \times (2.12)^2}{1,400 \times 0.0106} = 0.36 \text{ year} = 132 \text{ days.}$$

In 132 days the settlement is 75 percent of 0.76 foot or 6.8 inches.

Example 4.—Case 5 pressure distribution; type D loading; boundary condition B. All other conditions

are the same as in example 3 except that in this case the rigid footing is a loaded strip 10 feet wide and of relatively great length instead of a circular bearing area and consolidation caused by weight of sand is complete prior to loading. Figure 11 shows a vertical cross section. To find p_z at the top of the compressible layer and at a point on the vertical center axis through the cross section, apply equation (11). The angle, α , is expressed in radians and $\sin \alpha = 0.80$.

$$p_z = \frac{p}{\pi}(\alpha + \sin \alpha) = \frac{6,000}{3.14}(0.93 + 0.80) = 3,290 \text{ pounds per square foot.}$$

$$\text{At the lower boundary and on the centerline, } p_z = \frac{6,000}{3.14}(0.64 + 0.60) = 2,375 \text{ pounds per square foot.}$$

This is case 5 pressure distribution, and $u = 1.38$. The average voids ratio prior to loading is the average of e corresponding to 1,000 and of e corresponding to 1,500 pounds per square foot and is $1.21 = e_1$. Subsequent to loading the values of e at the top and bottom of the compressible stratum correspond to the equilibrium pressures, 4,290 and 3,875 pounds per square foot, respectively. From figure 2 these values of e are 0.90 and 0.92, the average being $0.91 = e_2$. The total settlement is therefore

$$Q = \left[\frac{1.21 - 0.91}{1 + 1.21} \right] \times 5 = 0.68 \text{ foot or 8.2 inches.}$$

To compute the rate of consolidation, apply equations (24) and (26) and make use of tables 2 and 3. $J' = 0.87$ for $u = 1.38$ (by interpolation). For $e = \frac{e_1 + e_2}{2} = 1.06$, $c = 0.0134$.

$$\text{For } q_1 = q_3 = q_5 = 25 \text{ percent, } t = \frac{N_5 D_0^2}{1,400c} = \frac{0.11 \times (2.26)^2}{1,400 \times 0.0134} = 0.03 \text{ year} = 11 \text{ days.}$$

$$\text{For } q_5 = 50 \text{ percent, } t = \frac{0.46 \times (2.26)^2}{1,400 \times 0.0134} = 0.125 \text{ year} = 46 \text{ days.}$$

$$\text{For } q_5 = 75 \text{ percent, } t = \frac{1.14 \times (2.26)^2}{1,400 \times 0.0134} = 0.310 \text{ year} = 113 \text{ days.}$$

In 113 days the strip settles 75 percent of 8.2 inches or 6.2 inches.

The preceding examples are intended to illustrate the application of the theory of soil consolidation. Part 2 contains the derivation of equations and the development of the basic theory.

PART 2.—DEVELOPMENT OF THEORY OF SOIL CONSOLIDATION

In 1885, J. Boussinesq⁶ developed in considerable detail and usable form a solution of the general problem of determining the distribution of stresses and strains in a semi-infinite, elastic, isotropic solid, bounded by a plane and loaded by a single concentrated force at a point on that plane. The formulas of Boussinesq have had wide application in the development of the science of soil mechanics.

The distribution of stresses across any plane parallel to the plane surface is shown in figure 12. If a single concentrated vertical load, P , is applied at a point on the surface, then at a radial distance, R , from this point and at a depth, z , below the surface, the stresses at the point (x, y) are:

$$p_R = \frac{3P}{2\pi R^2} \frac{z}{R}, \text{ parallel to } R \quad (32)$$

$$p_x = \frac{3P}{2\pi R^2} \frac{zx^2}{R^3}, \text{ parallel to } x \text{ axis} \quad (33)$$

$$p_y = \frac{3P}{2\pi R^2} \frac{zy^2}{R^3}, \text{ parallel to } y \text{ axis} \quad (34)$$

$$p_z = \frac{3P}{2\pi R^2} \frac{z^3}{R^3}, \text{ parallel to } z \text{ axis} \quad (35)$$

The derivation of these equations may be found in almost any standard text dealing with the theory of elasticity.⁷ These equations contain no elastic constants, however, and for this reason it has probably been assumed that the distribution of stresses does not depend upon the type of material. Cummings,⁸ Krynine,⁹ and others have discussed this point at length.

A complete analysis of the stress distribution including planes not parallel to the boundary involves 6 components of stress, 3 shears and 3 normal stresses, and the elastic constants appear in some of these equations. In the theory of consolidation of soils we are concerned mainly with the vertical normal stress, p_z . At a given depth, z , it is seen from equation (35) that p_z is greatest when R is smallest, that is, the maximum values for p_z are along the vertical centerline of the loaded area ($x=y=0$, fig. 12).

The value of p_z at any point is easily computed, knowing that $R^2 = r^2 + z^2 = x^2 + y^2 + z^2$. If a combination of point loads is given, we may obtain the stresses at a point resulting from each load considered separately, then add these and obtain the total pressure at the point produced by the combination of loads. Gilboy¹⁰ has shown that by dividing a uniformly loaded area into rectangular elements so that the longer side of any element is less than half the distance from the center of the area to the point, the computed value for p_z at the point will not deviate from the correct value by more than 6 percent. The accuracy is greater with further subdivision of the rectangular elements.

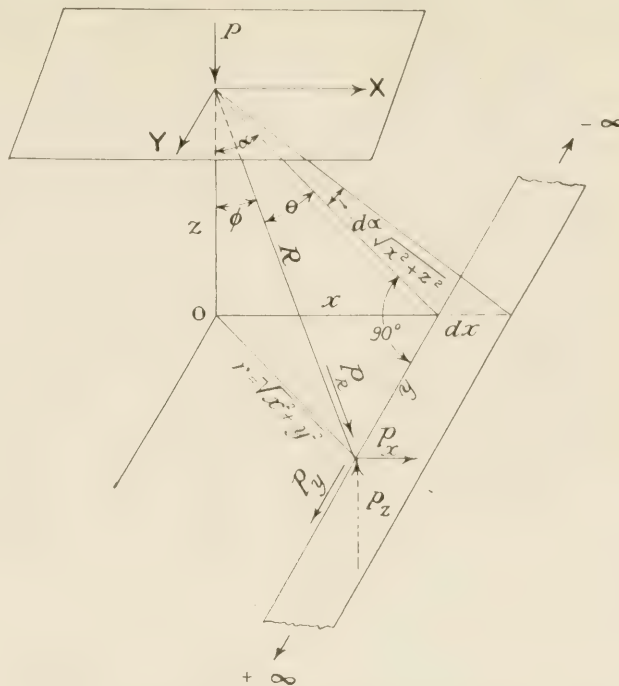


FIGURE 12.—DISTRIBUTION OF STRESSES ACROSS ANY PLANE PARALLEL TO THE PLANE SURFACE.

FORMULAS DEVELOPED FOR DETERMINING PRESSURE AT POINT BELOW LOAD

With reference to figure 12, if the expression for $p_z = \frac{3z^3P}{2\pi R^5}$ is integrated in the y direction from $y = -\infty$ to $y = +\infty$ and the resulting "line load" is multiplied by dx , we obtain the total load on a strip infinitely long and of width, dx , produced by the point load, P , at the boundary. In this integration, x and z are constant. Thus:

$$p_z = \frac{3Pz^3}{2\pi R^5} = \frac{3Pz^3}{2\pi(x^2 + y^2 + z^2)^{5/2}}$$

$$\int p_z = dx \frac{3Pz^3}{\pi} \int_0^{\pi} \left[\frac{x^2 + y^2 + z^2}{x^2 + z^2} \right]^{-5/2} (x^2 + z^2)^{-5/2} dy$$

This is easily integrated if we put $\frac{y}{(x^2 + z^2)^{1/2}} = \tan \theta$ (see fig. 12). Then

$$\int p_z = (dx) \frac{3Pz^3}{\pi(x^2 + z^2)^2} \int_{\theta=0}^{\theta=\pi/2} \cos^3 \theta d\theta,$$

or

$$\int p_z = \frac{3Pz^3}{\pi(x^2 + z^2)^2} \left[\sin \theta - \frac{\sin^3 \theta}{3} \right]_0^{\pi/2} dx = \frac{2Pz^3}{\pi(x^2 + z^2)^2} dx \quad (36)$$

If this expression is again integrated in the x direction, taking twice the value obtained between the limits, $x=0$ and $x=\infty$ (fig. 12), the result is equal to P , the applied force. Obviously, on any plane of infinite extent and at a finite distance below and parallel to the loading surface, the total distributed load is equal to the applied load at the surface. The mathematical result is in agreement with this self-evident fact.

⁶ Application des Potentiels a l'Etude de l'Equilibre et du Mouvement des Solides Elastiques. Paris, 1885.

⁷ For example, see chapter 11 of Theory of Elasticity, by S. Timoshenko.

⁸ Distribution of Stresses under a Foundation, by A. E. Cummings, Proceedings, American Society of Civil Engineers, vol. 61, no. 6, August 1935.

⁹ Discussion of Cummings' paper by Clement C. Williams, D. P. Krynine, and L. C. Wilcoxon, Proceedings, American Society of Civil Engineers, vol. 68, no. 8, pt. I, October 1935.

¹⁰ Earths and Foundations. Progress Report of Special Committee, Proceedings, American Society of Civil Engineers, vol. 59, no. 5, May 1933.

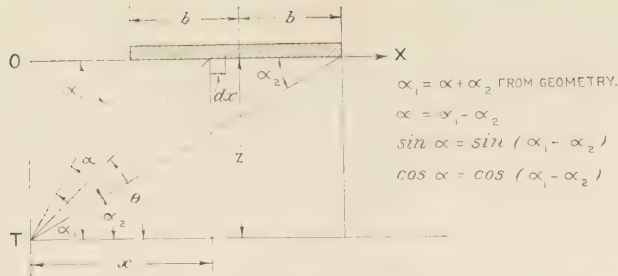


FIGURE 13.—VERTICAL CROSS SECTION OF A UNIFORMLY LOADED AREA OF INFINITE LENGTH AND OF WIDTH 2*b*.

In equation (36), if *P* is taken as *p*, the vertical pressure per unit area, and if *p* is considered as concentrated at the center of a unit area, then the total vertical pressure on the strip at a distance below the surface is equal to $\frac{2pz^3}{\pi(x^2+z^2)^2} dx$. This is the total pressure, $\int p_z$, on a strip below the surface produced by a unit load on the surface. Conversely, the total load on an infinitely long strip at the surface and parallel to the *y* axis, produced by applying the unit load, *p*, over the length of this strip will cause a vertical pressure $\frac{2pz^3}{\pi(x^2+z^2)^2} dx$, at a point below the surface. Briefly, a point load at the surface produces a total load, $\frac{2pz^3}{\pi(x^2+z^2)^2} dx$ on a strip below the surface and conversely, a strip load of unit load per unit area on the surface produces a pressure, $\frac{2pz^3}{\pi(x^2+z^2)^2} dx$ at a point below the surface, action and reaction being equal. It will be noted that the integrations of the Boussinesq formulas are with reference to areas or points below the loaded area at the surface. At the surface, *z*=0 and there is nothing to integrate that has reference to the fundamental equation (35).

Figure 13 is a vertical cross section of a uniformly loaded area (*p* per unit of area) of infinite length and of width, 2*b*. The load on a strip of infinitesimal width, *dx*, produces a pressure, *p_z*, at a point T equal to $\frac{2pz^3}{\pi(x^2+z^2)^2} dx$. If this expression is integrated between limits, there is obtained the value for *p_z* at T resulting from the total load over the whole area, infinite in length and of width, 2*b*. The integration is more easily performed if we let $\frac{x}{z} = \cot \theta$ (fig. 13). Then at the point T,

$$p_z = \frac{2p}{\pi z} \int_{x=2b}^x \left(1 + \frac{x^2}{z^2}\right)^{-2} dx = -\frac{2p}{\pi} \int_{\alpha_1}^{\alpha_2} (1 + \cot^2 \theta)^{-2} \csc^2 \theta d\theta$$

$$= \frac{p}{\pi} [\alpha - \sin \alpha \cos(\alpha_1 + \alpha_2)] \dots \dots \dots (37)$$

If the point, T, is on the vertical centerline through the vertical cross section (fig. 13), then $\alpha_2 = 180^\circ - \alpha_1$ and equation (37) reduces to

$$p_z = \frac{p}{\pi} (\alpha + \sin \alpha) \dots \dots \dots (38)$$

For uniform pressure distribution (*p* per unit area) over a circular bearing area of radius *r* (see fig. 14) the

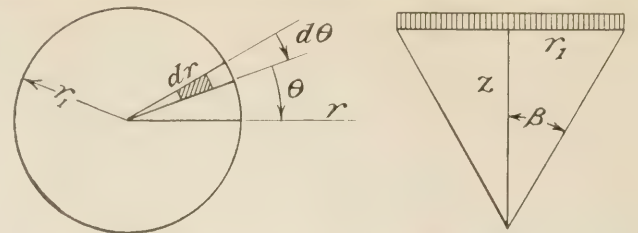


FIGURE 14.—CIRCULAR LOADED AREA.

pressure at a point below the loaded area and on the vertical centerline is,

$$p_z = \int_0^{r_1} \int_0^{2\pi} \frac{3p}{2\pi} \frac{z^3}{(z^2+r^2)^{5/2}} r dr d\theta = 3pz^3 \int_0^{r_1} \frac{r dr}{(r^2+z^2)^{5/2}}$$

Integrating and simplifying,

$$p_z = p \left[1 - \frac{1}{\left(1 + \frac{r_1^2}{z^2}\right)^{3/2}} \right] \text{ and since } \frac{r_1}{z} = \tan \beta$$

$$p_z = p(1 - \cos^3 \beta) \dots \dots \dots (39)$$

MECHANICS OF SOIL CONSOLIDATION OUTLINED

Consider the flow or egress of water from a wet, compressible, soil stratum subjected to vertical loading. The soil is relatively impervious and is "sandwiched" between two permeable sand-gravel layers, L, L, figure 15. The movement of water saturating the stratum is in both the upward and downward directions, parallel to the vertical axis, ZZ, and directed toward the two permeable layers, L, L.

At any finite time, *t*, after load application, that part of the total pressure carried by the water, *p_w*, is considered as the "hydrodynamic excess."¹¹ The remainder of the load is carried by the solid material, the "soil skeleton." At the outset of loading (*t*=0) all of the pressure is carried by water if the compressible layer is saturated. At any subsequent time, the hydrodynamic excess, *p_w*, is zero at the surfaces of contact of the soil stratum and the permeable layers, but elsewhere it is a finite quantity, varying with the depth, *z*. The upper and lower faces, AB and CD of the elementary prism, ABCD, are each of unit area. The thickness of the elementary prism is Δ*z* (fig. 15).

We may then define the following expressions:

p=the total pressure per unit cross-sectional area. It includes the pressure on the soil skeleton and the pressure, *p_w*, carried by water alone.

p_w=the pressure per unit cross-sectional area that is carried by water alone.

p - p_w=the pressure per unit cross-sectional area that is carried only by the soil particles, i. e., by the soil skeleton.

a=the rate of change of the voids ratio with respect to (*p - p_w*) and is called the coefficient of compressibility.

k₀=the reduced coefficient of permeability; see equation (4), Part 1.

$\frac{dp_w}{dz}$ =the rate of change of the hydraulic pressure, *p_w*, with respect to the depth, *z*. It is the hydraulic pressure gradient.

$\frac{d^2 p_w}{dz^2}$ =the rate of change of $\frac{dp_w}{dz}$ with respect to *z* and

¹¹ Terzaghi, K., *Erdbaumechanik*. Franz Deuticke, Vienna, 1925.

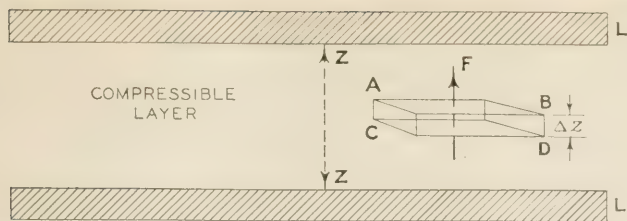


FIGURE 15.—ELEMENTARY PRISM OF COMPRESSIBLE MATERIAL BETWEEN TWO PERMEABLE LAYERS. F INDICATES DIRECTION OF FLOW.

is taken as constant through the very small vertical distance, Δz .

$\Delta z \frac{d^2 p_w}{dz^2}$ = the total change of $\frac{dp_w}{dz}$ within the interval, Δz .

$k_0 \frac{d^2 p_w}{dz^2} \Delta z$ = the rate at which the elementary prism loses water. It represents the difference between the quantity of water flowing from the upper face, AB, in unit time and the quantity flowing into the prism through the face, CD, in unit time. It is therefore the rate of volume reduction of the elementary prism.

$\frac{dp_w}{dt} = -\frac{d(p-p_w)}{dt}$ = the rate of change of $p-p_w$ with respect to time.

$a \frac{dp_w}{dt}$ = the rate of change in voids ratio with respect to time.

$a \frac{dp_w}{dt} \Delta z$ = the rate of change in volume of the elementary prism with respect to time.

The expressions $\frac{dp_w}{dz}$, $\frac{d^2 p_w}{dz^2}$, and $\frac{dp_w}{dt}$ are considered as the partial derivatives. The faces, AB and CD, of the elementary prism are both of unit horizontal cross-sectional area and are parallel to each other.

The amount of water lost by the elementary prism in unit time is $k_0 \frac{d^2 p_w}{dz^2} \Delta z$, which is the difference between the amount of water flowing out from the face AB and that flowing into the prism through the face CD. But the rate of loss of water (which is the rate of reduction in volume) by the elementary prism is also equal to $a \frac{dp_w}{dt} \Delta z$. Then $k_0 \frac{d^2 p_w}{dz^2} \Delta z = a \frac{dp_w}{dt} \Delta z$. Divide

both sides of this expression by a and write c for $\frac{k_0}{a}$.

Then, since Δz cancels,

$$c \frac{d^2 p_w}{dz^2} = \frac{dp_w}{dt} \quad (40)$$

This is analogous to the development of the equation for the flow of heat. A more complete derivation of equation (40) has been published.¹²

TIME-CONSOLIDATION RELATIONSHIPS FOR UNIFORM PRESSURE DISTRIBUTION DERIVED

A solution of equation (40) is

$$p_w = e^{-cK^2 t} (A \cos Kz + B \sin Kz) \quad (41)$$

where A , B , and K are constants.

¹² Mathematical Theory of the Process of Consolidation of Mud Deposits, by Alberto Ortenblad. Journal of Mathematics and Physics, vol. 9, no. 2.

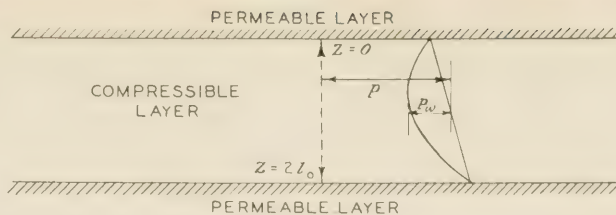


FIGURE 16.—PRESSURE DISTRIBUTION IN A COMPRESSIBLE LAYER BETWEEN TWO PERMEABLE LAYERS. AT ANY LEVEL, Z , p_w = THE PRESSURE TAKEN BY WATER. p = THE TOTAL PRESSURE. $p-p_w$ = THE PRESSURE ON THE SOIL SKELETON.

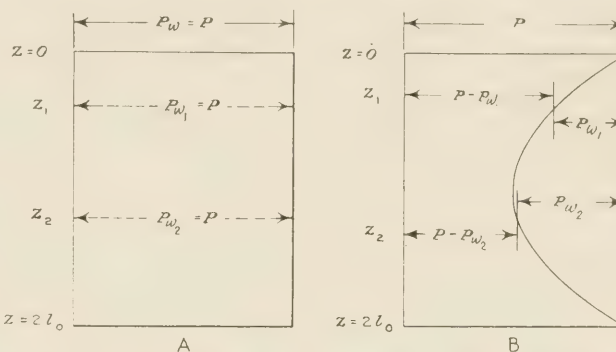


FIGURE 17.—A, PRESSURE CONDITIONS IN COMPRESSIBLE LAYER. AT THE INSTANT LOAD IS APPLIED ($t=0$), $p_w=p$ AT ALL LEVELS. B, PRESSURE CONDITIONS IN COMPRESSIBLE LAYER SUBSEQUENT TO LOAD APPLICATION. WHEN $t>0$, $p_w=0$ AT $z=0$ AND AT $z=2l_0$; $p_w < p$ AT ALL LEVELS.

In figure 16, the reduced thickness (the thickness the compressible stratum would have were there no voids between the particles) of the soil layer is $2l_0$ where l_0 is the maximum distance that water travels if the layer has a thickness, d_0 . That is, $d_0=2l_0$. Water saturates the soil. Apply load. At the very instant that load is applied, as noted before, the water in the soil takes all the pressure (see fig. 17-A). That is, at $t=0$, the total pressure, p , is equal to p_w at any depth, z (see fig. 17-A). At any other time after loading, p_w is less than the total pressure, some of which has been transferred to the soil skeleton. The condition, subsequent to applying load, is represented by figure 17-B. At any level, z , a part of the load, p_w , designated as the "hydrodynamic excess," is carried by water. The load carried by the soil skeleton at depth, z , is the total pressure minus p_w .

At any time subsequent to $t=0$, p_w is equal to 0 when $z=0$. The water escapes instantaneously at $z=0$.

The same condition prevails at the lower boundary, that is at $z=2l_0$, $p_w=0$ if t is greater than 0. At $t=0$, p_w is a function of z .

For present purposes, assume that the total pressure, p , (that on solid and that on water taken together) is uniform and constant throughout the depth, z .

We now consider equation (41) under the following boundary conditions.

$$p_w = (A \cos Kz + B \sin Kz) e^{-cK^2 t} \quad (41)$$

Condition 1: $p_w=0$ when $z=0$ and $t>0$.

The sine of 0 is 0 and $\sin Kz=0$.

Therefore, since $p_w=0$ and $B \sin Kz=0$, it follows that $A \cos Kz$ must also be 0. But $\cos 0=1$ and $A \cos 0=A$. Then A must equal zero. So condition 1 reduces equation (41) to

$$p_w = (B \sin Kz) e^{-cK^2 t} \quad (42)$$

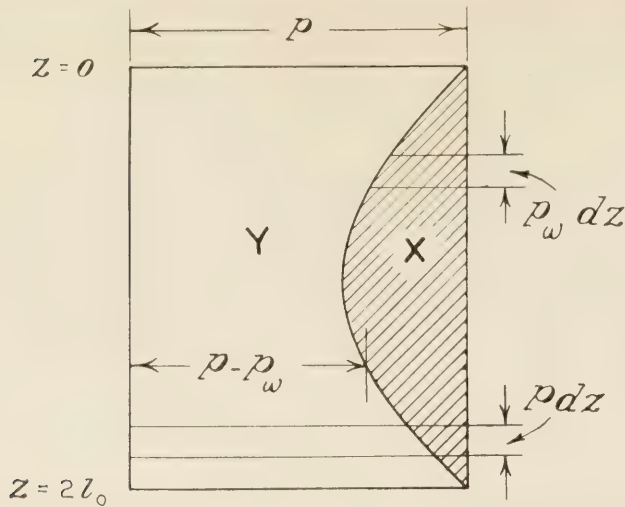


FIGURE 18.—FRACTIONS OF LOAD CARRIED BY SOIL SKELETON AND BY WATER.

Condition 2: When $z=2l_0$ and $p_w=0$, then $\sin Kz = \sin 2Kl_0 = 0$.

Hence Kz must be an integral multiple of π .

$z=2l_0$ and $Kz=2Kl_0$ for condition 2, and $\sin Kz=0$ when $Kz=2Kl_0=\pi, 2\pi, 3\pi \dots n\pi$, where n is any integer from 1 to infinity.

Since $2Kl_0 = n\pi$, $K = \frac{\pi}{2l_0} = \frac{2\pi}{2l_0} = \frac{3\pi}{2l_0} = \frac{n\pi}{2l_0}$.

The expression, e^{-Kz} , becomes $e^{-\frac{n^2\pi^2 ct}{4l_0^2}}$. Substitute for K the successive values, $\frac{\pi}{2l_0}, \frac{2\pi}{2l_0}, \frac{n\pi}{2l_0}$, etc. in equation (42), add and obtain:

$$p_w = a_1 e^{-\frac{\pi^2 ct}{4l_0^2}} \sin \frac{\pi z}{2l_0} + a_2 e^{-\frac{4\pi^2 ct}{4l_0^2}} \sin \frac{2\pi z}{2l_0} + \dots + a_n e^{-\frac{n^2\pi^2 ct}{4l_0^2}} \sin \frac{n\pi z}{2l_0} \dots (43)$$

When $t=0$, equation (43) becomes:

$$p_w = p = a_1 \sin \frac{\pi z}{2l_0} + a_2 \sin \frac{2\pi z}{2l_0} + a_3 \sin \frac{3\pi z}{2l_0} + \dots + a_n \sin \frac{n\pi z}{2l_0} \dots (44)$$

This is true for all values of z between 0 and $2l_0$. To find a_1 , multiply through by $\sin \frac{\pi z}{2l_0}$ and integrate all terms between the limits, 0 and $2l_0$. Then, as usually happens in the case of a Fourier series, all terms on the right, except the first, vanish, and since $p_w = a$ constant $= p$ when $t=0$,

then $p \int_0^{2l_0} \sin \frac{\pi z}{2l_0} dz = a_1 \int_0^{2l_0} \sin^2 \frac{\pi z}{2l_0} dz \dots (45)$

$a_1 = \frac{4p}{\pi}, a_2 = 0, a_3 = \frac{4p}{3\pi}, a_n = \frac{4p}{n\pi}$,

where n is odd.

Substituting the values of $a_1, a_2, a_3, \dots a_n$ in equation (43), there results

$$p_w = \frac{4p}{\pi} e^{-\frac{\pi^2 ct}{4l_0^2}} \sin \frac{\pi z}{2l_0} + \frac{4p}{3\pi} e^{-\frac{9\pi^2 ct}{4l_0^2}} \sin \frac{3\pi z}{2l_0} + \dots + \frac{4p}{(2n+1)\pi} e^{-\frac{(2n+1)^2\pi^2 ct}{4l_0^2}} \sin \frac{(2n+1)\pi z}{2l_0} \dots (46)$$

Where n is 0, 1, 2, 3, etc.

Multiply equation (46) through by dz and integrate between the limits, $z=0$ and $z=2l_0$. We have:

$$\int_0^{2l_0} p_w dz = \frac{4p}{\pi} e^{-\frac{\pi^2 ct}{4l_0^2}} \int_0^{2l_0} \sin \frac{\pi z}{2l_0} dz + \frac{4p}{3\pi} e^{-\frac{9\pi^2 ct}{4l_0^2}} \int_0^{2l_0} \sin \frac{3\pi z}{2l_0} dz + \dots + \frac{4p}{(2n+1)\pi} e^{-\frac{(2n+1)^2\pi^2 ct}{4l_0^2}} \int_0^{2l_0} \sin \frac{(2n+1)\pi z}{2l_0} dz \dots (47)$$

or $\int_0^{2l_0} p_w dz = -\frac{8l_0 p}{\pi^2} e^{-\frac{\pi^2 ct}{4l_0^2}} \left[\cos \frac{\pi z}{2l_0} \right]_0^{2l_0} - \frac{8pl_0}{9\pi^2} e^{-\frac{9\pi^2 ct}{4l_0^2}} \left[\cos \frac{3\pi z}{2l_0} \right]_0^{2l_0} \dots - \frac{8pl_0}{(2n+1)^2\pi^2} e^{-\frac{(2n+1)^2\pi^2 ct}{4l_0^2}} \left[\cos \frac{(2n+1)\pi z}{2l_0} \right]_0^{2l_0} = \frac{16pl_0}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} e^{-\frac{(2n+1)^2\pi^2 ct}{4l_0^2}}$

This is the total p_w at any time greater than zero.

The total p_w at $t=0 = \int_0^{2l_0} p dz = p[z]_0^{2l_0} = 2l_0 p$

since $p_w = p$ when $t=0$.

$\int_0^{2l_0} p_w dz =$ total p_w at any subsequent time, t .

$1 - \frac{\int_0^{2l_0} p_w dz}{2l_0 p} = q_1$, the percentage of consolidation at time, t .

That is

$$q_1 = 1 - \frac{16pl_0}{\pi^2} \left(e^{-N} + \frac{e^{-9N}}{9} + \frac{e^{-25N}}{25} + \frac{e^{-49N}}{49} + \dots \right) \frac{1}{2l_0 p}$$

or $q_1 = 1 - 8/\pi^2 \left(e^{-N} + \frac{e^{-9N}}{9} + \frac{e^{-25N}}{25} + \frac{e^{-49N}}{49} + \dots \right) \dots (48)$

where $N = \frac{\pi^2 ct}{4l_0^2}$

Figure 18 illustrates diagrammatically the load fractions carried by the clay skeleton and the water.

In figure 19, $X =$ load fraction carried by water, and $Y =$ load fraction carried by soil.

The total $p_w = X = \int_0^{2l_0} p_w dz$ and $X + Y = \int_0^{2l_0} p dz =$ the total pressure.

$q_1 = 1 - \frac{\int_0^{2l_0} p_w dz}{\int_0^{2l_0} p dz} = \frac{Y}{X+Y}$ when $t=t_1 > 0$

$q_1 = 1 - 8/\pi^2 \left(e^{-\frac{\pi^2 ct}{4l_0^2}} + 9e^{-\frac{9\pi^2 ct}{4l_0^2}} + \dots + \frac{1}{(2n+1)^2} e^{-\frac{(2n+1)^2\pi^2 ct}{4l_0^2}} \right)$

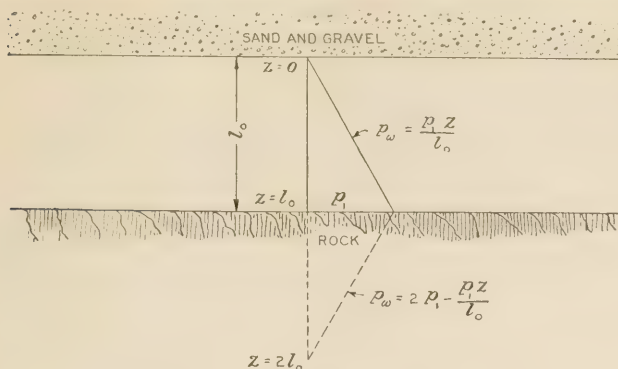


FIGURE 19.—TRIANGULAR PRESSURE DISTRIBUTION. THE LOWER TRIANGLE IS THE MIRROR IMAGE OF THE UPPER ONE.

DERIVATION OF TIME-CONSOLIDATION RELATIONSHIP FOR TRIANGULAR PRESSURE DISTRIBUTION, PRESSURE INCREASING WITH DEPTH

For a triangular pressure distribution (see fig. 19) the two boundary conditions,

Condition 1: $p_w=0$ at $z=0$

and condition 2: $\frac{dp_w}{dz}=0$ at $z=l_0$

must satisfy equation (41),

$$p_w = e^{-\kappa^2 ct} (A \cos Kz + B \sin Kz).$$

From condition 1, this equation reduces to equation (42) as before and

$$p_w = e^{-\kappa^2 ct} (B \sin Kz) \dots \dots \dots (42)$$

For the second condition, at $z=l_0$,

$$\frac{dp_w}{dz} = e^{-\kappa^2 ct} BK \cos Kz = 0 = e^{-\kappa^2 ct} BK \cos Kl_0.$$

But

$$\cos Kl_0 = 0 \text{ when } Kl_0 = \frac{m\pi}{2}$$

where m is any odd integer. That is,

$$K = \frac{\pi}{2l_0}, \frac{3\pi}{2l_0}, \frac{5\pi}{2l_0}, \dots, \frac{m\pi}{2l_0}.$$

Substitute these values of K successively in (42) and add the results together; we thus obtain

$$p_w = a_1 e^{-\frac{\pi^2 ct}{4l_0^2}} \sin \frac{\pi z}{2l_0} + a_2 e^{-\frac{9\pi^2 ct}{4l_0^2}} \sin \frac{3\pi z}{2l_0} + \dots$$

to infinity \dots \dots \dots (49)

At $t=0$, equation (49) becomes

$$p_w = a_1 \sin \frac{\pi z}{2l_0} + a_2 \sin \frac{3\pi z}{2l_0} + \dots + a_m \sin \frac{m\pi z}{2l_0} \dots (50)$$

where m is 1, 3, 5, etc., i. e., any odd, positive integer.

The coefficient, a_m , is evaluated in the usual manner.

At $t=0$, $p_w=f(z)=\frac{p_1 z}{l_0}$ between the limits $z=0$ and $z=l_0$. At $t=0$, between the limits $z=l_0$ and $z=2l_0$,

$$p_w=f_1(z)=2p_1 - \frac{p_1 z}{l_0}.$$

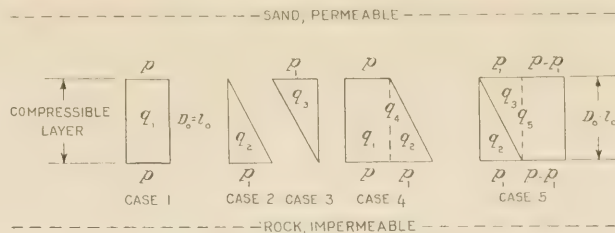


FIGURE 20.—TYPES OF PRESSURE DISTRIBUTION IN A COMPRESSIBLE LAYER.

Then

$$a_m = \frac{1}{l_0} \left[\int_0^{l_0} \frac{p_1 z}{l_0} \sin \frac{m\pi z}{2l_0} dz + \int_{l_0}^{2l_0} \left(2p_1 - \frac{p_1 z}{l_0} \right) \sin \frac{m\pi z}{2l_0} dz \right]$$

Where m is any odd, positive integer.

Integrating, $a_m = \frac{8p_1}{m^2 \pi^2}$ for $m=1, 5, 9, 13, 17$, etc.,

and

$$a_m = -\frac{8p_1}{m^2 \pi^2} \text{ for } m=3, 7, 11, 15, 19, \text{ etc.}$$

By substitution equation (49) becomes

$$p_w = \frac{8p_1}{\pi^2} e^{-\frac{\pi^2 ct}{4l_0^2}} \sin \frac{\pi z}{2l_0} - \frac{8p_1}{9\pi^2} e^{-\frac{9\pi^2 ct}{4l_0^2}} \sin \frac{3\pi z}{2l_0} \pm \dots \dots \dots (51)$$

Then $\int_0^{l_0} p_w dz = \frac{8p_1}{\pi^2} \sum_{m=1}^{\infty} \frac{(-1)^{\frac{m-1}{2}}}{m^2} e^{-\frac{m^2 \pi^2 ct}{4l_0^2}} \int_0^{l_0} \sin \frac{m\pi z}{2l_0} dz$

$$= \frac{16p_1 l_0}{\pi^3} e^{-\frac{\pi^2 ct}{4l_0^2}} - \frac{16p_1 l_0}{27\pi^3} e^{-\frac{9\pi^2 ct}{4l_0^2}} \pm \dots \frac{16p_1 l_0^2}{(2n+1)^3 \pi^3} e^{-\frac{(2n+1)^2 \pi^2 ct}{4l_0^2}}$$

Where $n=0, 1, 2, 3$, etc.

At $t=0$, $f(z) = \frac{p_1 z}{l_0}$ and $\int_0^{l_0} f(z) dz = \frac{p_1 l_0}{2}$

$$q_2 = 1 - \frac{\int_0^{l_0} p_w dz}{\int_0^{l_0} f(z) dz}$$

$$= 1 - \frac{32}{\pi^3} \left(e^{-\frac{\pi^2 ct}{4l_0^2}} - \frac{1}{27} e^{-\frac{9\pi^2 ct}{4l_0^2}} + \frac{1}{125} e^{-\frac{25\pi^2 ct}{4l_0^2}} - \dots \right)$$

or writing $N = \frac{\pi^2 ct}{4l_0^2}$,

$$q_2 = 1 - \frac{32}{\pi^3} \left(e^{-N} - \frac{1}{27} e^{-9N} + \frac{1}{125} e^{-25N} - \dots \right) \dots (52)$$

FORMULAS APPLICABLE TO OTHER TYPES OF PRESSURE DISTRIBUTION DEVELOPED

General relations.—Figure 20 illustrates five different types of pressure distribution. In this figure the conditions illustrated are for a compressible stratum overlaid with sand and resting on impervious rock.

The percentages of consolidation for the five types of pressure distribution shown in figure 21 are expressed as follows:

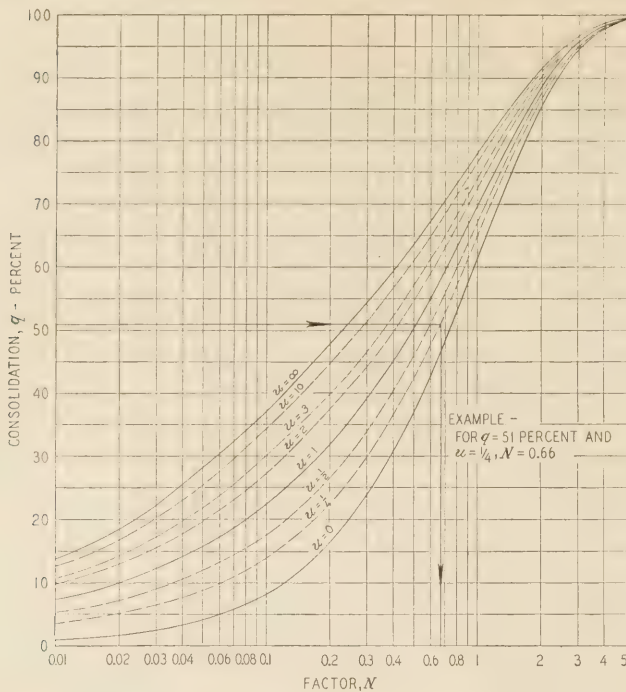


FIGURE 21.—VALUES OF q AND N FOR DIFFERENT VALUES OF u .

Case 1.—Uniform or rectangular pressure distribution.

$$q_1 = 1 - \frac{8}{\pi^2} \left(e^{-N} + \frac{1}{9} e^{-9N} + \frac{1}{25} e^{-25N} + \dots \right)$$

Case 2.—Triangular pressure distribution, pressure = 0 at upper surface.

$$q_2 = 1 - \frac{32}{\pi^3} \left(e^{-N} - \frac{1}{27} e^{-9N} + \frac{1}{125} e^{-25N} - \dots \right)$$

Case 3.—Triangular pressure distribution, pressure = 0 at lower surface.

$$q_3 = \frac{p \left(q_1 - \frac{q_2}{2} \right)}{\frac{p}{2}} = 2q_1 - q_2$$

Case 4.—Trapezoidal pressure distribution, pressure increasing with depth.

$$q_4 = \frac{pq_1 + q_2 \frac{p_1}{2}}{p + \frac{p_1}{2}}$$

Case 5.—Trapezoidal pressure distribution, pressure decreasing with depth.

$$q_5 = \frac{pq_1 - \frac{p_1}{2} q_2}{p - \frac{p_1}{2}} = \frac{(p - p_1) q_1 + \frac{p_1}{2} q_3}{p - \frac{p_1}{2}}$$

If the compressible layer is in contact with permeable sand layers above and below, then equation (48) holds for all of the five types of pressure distribution shown in figure 20 and interpolation formulas are not needed

in this comparatively simple case for, if q_1 , the percentage consolidation, is plotted against the time, the curves for the different pressure distributions will theoretically coincide for the same soil. The ultimate amount of consolidation may vary with the type of pressure distribution but the percentage of the ultimate consolidation attained at the end of a definite period of time is theoretically independent of the pressure or its manner of distribution if the relatively impervious stratum is confined between two relatively permeable layers, and the rate at which q_1 increases depends solely on the permeability, compressibility, and thickness of the clay layer.

The expressions for cases 4 and 5, trapezoidal pressure distributions, are combinations of equations (48) and (52) and are obtained by integration according to the procedure followed in deriving equation (52). The expressions are, case 4,

$$q_4 = \frac{pq_1 + \frac{p_1}{2} q_2}{p + \frac{p_1}{2}} \dots \dots \dots (53)$$

and case 5,

$$q_5 = \frac{pq_1 - \frac{p_1}{2} q_2}{p - \frac{p_1}{2}} \dots \dots \dots (54)$$

Case 3, triangular pressure distribution, u being infinite, is a special case of equation (54). For case 3, $p = p_1$ and hence from equation (54),

$$q_3 = 2q_1 - q_2 \dots \dots \dots (55)$$

When the expressions on the right of equations (48) and (52) are substituted for q_1 and q_2 , equation (55) is written

$$q_3 = 2 - \frac{16}{\pi^2} \left[e^{-N} + \frac{1}{9} e^{-9N} + \frac{1}{25} e^{-25N} + \dots \right] - 1 + \frac{32}{\pi^3} \left[e^{-N} - \frac{1}{27} e^{-9N} + \frac{1}{125} e^{-25N} - \dots \right] \dots (56)$$

Formulas for trapezoidal pressure distributions.—With reference to figure 20, both q_4 and q_5 approach q_1 as a limit as p_1 approaches the limit zero. This is also evident from equations (53) and (54). Note also that $p + \frac{p_1}{2}$ is the area of the trapezoid, case 4, when $l_0 = 1$ and $p - \frac{p_1}{2}$ is the area of the trapezoid, case 5, for $l_0 = 1$.

For the condition, $q_1 = q_2 = q_3$, the numerical value, N_4 , will be between that for N_1 and N_2 , that is,

$$N_4 = N_1 + J(N_2 - N_1).$$

This is equation (25), Part 1. Then

$$(1 - q_1) = (1 - q_2) = (1 - q_3) \text{ and } \log_e (1 - q_1) = \log_e (1 - q_2) = \log_e (1 - q_3) = \log_e R.$$

Neglecting all terms in e except the first in equations (48), (52), and (53),

$$\log_e (1 - q_1) = \log_e \frac{8}{\pi^2} - N_1 = \log_e R \dots \dots \dots (57)$$

$$\log_e (1 - q_2) = \log_e \frac{32}{\pi^3} - N_2 = \log_e R \quad (58)$$

$$\log_e (1 - q_4) = \log_e \left[\frac{\frac{8}{\pi^3} p + \frac{16}{\pi^3} p_1}{p + \frac{p_1}{2}} \right] - N_4 = \log_e R \quad (59)$$

where \log_e denotes the natural logarithm. Equations (57), (58), and (59) are easily solved for N_1 , N_2 , and N_4 and the values are substituted in the expression,

$$J = \frac{N_4 - N_1}{N_2 - N_1} \quad (60)$$

which is equation (25) solved for J . Thus

$$J = \frac{\log \left[\frac{2p\pi + 4p_1}{\pi(2p + p_1)} \right]}{\log \frac{4}{\pi}} \quad (61)$$

Since for case 4, $u = \frac{p}{p + p_1}$ or $p = \frac{p_1 u}{1 - u}$, then by substitution in equation (61) one obtains

$$J = \frac{\log \left[\frac{2(\pi - 2)u + 4}{\pi(1 + u)} \right]}{\log \frac{4}{\pi}} \quad (62)$$

which is equation (21), Part 1.

For the condition, $q_1 = q_3 = q_5$, one may write

$$J' = \frac{N_5 - N_3}{N_1 - N_3} \quad (63)$$

Equation (63) is equation (26), Part 1, solved for J' . Furthermore, $1 - q_1 = 1 - q_3 = 1 - 2q_1 + q_2$, from equation (55), $= 1 - q_5$. By dropping all terms in e except the first,

$$\log_e (1 - q_1) = \log_e \frac{8}{\pi^2} - N_1 = \log_e R \text{ as before} \quad (57)$$

and

$$\begin{aligned} \log_e (1 - q_3) &= \log_e (1 - 2q_1 + q_2) - N_3 \\ &= \log_e \left[\frac{16}{\pi^2} - \frac{32}{\pi^3} \right] - N_3 = \log_e R \quad (64) \end{aligned}$$

In equation (54), $2q_1 - q_3$ may be substituted for q_2 to obtain

$$q_5 = \frac{(p - p_1)q_1 + \frac{p_1}{2} q_3}{p - \frac{p_1}{2}} \quad (65)$$

Then

$$\log_e (1 - q_5) = \log_e \left[\frac{(p - p_1) \frac{8}{\pi^2} + p_1 \left(\frac{8}{\pi^2} - \frac{16}{\pi^3} \right)}{p - \frac{p_1}{2}} \right] - N_5 \quad (66)$$

After substituting the values obtained by solving equations (57), (64), and (66) for N_1 , N_3 , and N_5 in equation (63) and substituting $\frac{p_1 u}{u - 1}$ for p , the expression obtained for J' is

$$J' = \frac{\log \left[\frac{u(\pi - 2) + 2}{(\pi - 2)(u + 1)} \right]}{\log \left[\frac{\pi}{2(\pi - 2)} \right]} \quad (67)$$

This is equation (22) of Part 1.

The expression for J was derived by Gilboy.¹³ The expression for J' was given by Kimball.¹⁴ Neither Gilboy nor Kimball indicated the derivations and both have given negative signs for equations (62) and (67). It would of course be impossible to obtain table 3 of Part 1 of this paper with negative values for J and J' . Since only the first terms in e were used in their derivations, the expressions for J and J' are of course only approximate.

Figure 21 shows curves obtained by plotting values for N on log scale as abscissae against corresponding values for q plotted as ordinates. The different curves are for 8 particular values assigned to u , these being $u = 0, \frac{1}{4}, \frac{1}{2}, 1, 2, 3, 10$, and ∞ . For intermediate values of u , values of q and N may be found by interpolation.

CHARACTERISTICS OF THEORETICAL TIME-CONSOLIDATION CURVE DISCUSSED

Equation (48) is one of a smooth curve having several important characteristics, one of which is illustrated in figure 22-A and 22-B. Figure 22-A is the curve obtained by taking $t = N$, that is, $\frac{\pi^2 c}{4l_0^2}$ as unity in equation (48). Let q_1 be the percentage consolidation at the end of a definite time interval, t , and t_1 the time required to produce half this percentage of consolidation, i.e., $\frac{1}{2} q_1$. For $q_1 = 80$ percent, figure 22-A, $t = 1.4$

minutes and the time, t_1 , required to effect $\frac{1}{2}$ of 80 percent consolidation is seen to be 0.3 minute from the time consolidation curve, figure 22-A. The ratios, $\frac{t}{t_1}$ are plotted against q_1 in figure 22-B. Within the range $q_1 = 0$ to $q_1 = 70$ percent, the ratio, $\frac{t}{t_1}$ is fairly constant and is approximately equal to 4. This indicates that within this range the time-consolidation curve is approximately a parabola. For values of q_1 greater than 70 percent, the ratios, $\frac{t}{t_1}$, increase. An ordinate value of $q_1 = 90$ percent on the theoretical curve, figure 22-A, corresponds to a value of $\frac{t}{t_1} = 5.27$, figure 22-B. The value, $t = 2.1$ minutes, corresponds to the value, $\frac{t}{t_1} = 5.27$.

Since the relation between q_1 and N_1 is fixed, the ratio, $\frac{t}{t_1} = 5.27$ for $q_1 = 90$ percent is invariant for all values assigned to $\frac{\pi^2 c}{4l_0^2}$.

As a rule the time-compression curves for relatively impermeable soils have very nearly the same shape as a theoretical curve up to a point which, on the theoretical curve, has an ordinate of 90 percent. Beyond this point the two curves diverge appreciably. Since an

¹³ See equation (31) in "Earths and Foundations," Proceedings, American Society of Civil Engineers, vol. 59, no. 5, May 1933.

¹⁴ Proceedings, American Society of Civil Engineers, vol. 59, no. 6, August 1933.

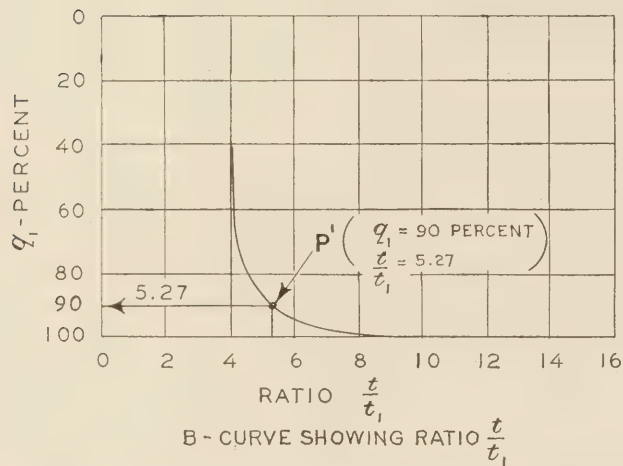
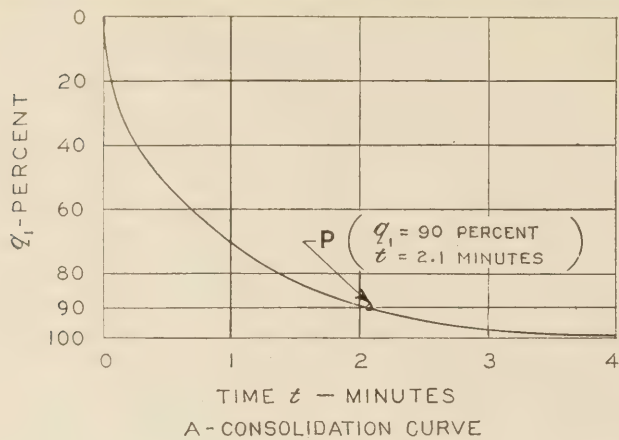


FIGURE 22.—RELATIONS BETWEEN PERCENTAGE OF CONSOLIDATION AND TIME, WHEN $t=N$.

ordinate of 90 percent on the theoretical curve corresponds to a definite value of $\frac{t}{t_1}=5.27$, it is easy to compute the coefficient of consolidation of a soil by adapting the actual curve to the theoretical one according to the following procedure:

1. Plot the laboratory time-consolidation curve, corresponding to figure 22-A.

2. From the time-consolidation curve plot the $\frac{t}{t_1}$ ratios corresponding to definite percentages of consolidation.

3. From the $\frac{t}{t_1}, q_1$ curve find the value of q_1 corresponding to $\frac{t}{t_1}=5.27$.

4. From the time-consolidation curve find the value of t corresponding to the value of q_1 for $\frac{t}{t_1}=5.27$. Call this particular value t_2 .

5. Substitute the value for t_2 thus found in the expression, $\frac{\pi^2 c}{4l_0^2} t_2=2.1$, and solve for c ,

$$c = \frac{2.1 \times 4l_0^2}{\pi^2 t_2} = \frac{0.85l_0^2}{t_2} \text{ ----- (68)}$$

In testing with the Terzaghi compression device, the sample is confined between two permeable layers and hence $l_0 = \frac{1}{2} d_0$. In this case,

$$c = \frac{0.85d_0^2}{4t_2} \text{ ----- (69)}$$

This is another method of computing c . [See equation (8).]

STATUS OF FEDERAL-AID HIGHWAY PROJECTS

AS OF FEBRUARY 28, 1937

STATE	APPORTIONMENT ¹		COMPLETED			UNDER CONSTRUCTION			APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR NEW PROJECTS
	Estimated Total Cost	Federal Aid	Estimated Total Cost	Miles	Federal Aid	Estimated Total Cost	Miles	Federal Aid	Estimated Total Cost	Miles	Federal Aid	
Alabama	\$ 7,872,980	\$ 25,800	\$ 911,981	9.0	\$ 455,990	\$ 911,981	36.9	\$ 801,590	\$ 400,780	\$ 6,990,410	46.5	
Arizona	5,294,661	1,508,964	1,449,258	111.1	1,087,452	1,449,258	51.8	288,184	204,801	2,593,444	6.6	
Arkansas	4,462,681	3,295,486	8,617,998	148.5	4,920,707	1,413,572	66.4	1,936,157	1,934,928	3,115,181	127.8	
California	6,911,198	2,030,922	2,999,222	150.4	1,967,481	4,920,707	219.9	2,137,988	1,228,852	5,021,846	42.8	
Colorado	2,388,339	388,949	709,328	14.2	599,556	1,967,481	103.6	995,170	953,516	2,689,278	49.4	
Connecticut	1,843,750	494,124	450,080	12.9	214,830	257,325	8.7	257,325	128,658	1,848,834	7.0	
Delaware	5,020,325	832,414	1,901,255	27.5	950,623	409,870	61.8	409,870	204,935	3,453,937	3.7	
Florida	9,569,722	1,145,317	2,552,647	86.2	1,276,308	791,880	133.0	791,880	395,940	7,361,754	41.9	
Georgia	4,635,991	2,698,759	866,009	56.1	518,151	751,776	39.9	751,776	455,870	2,091,240	39.9	
Idaho	15,564,720	8,317,693	4,575,896	134.5	2,253,474	4,302,725	122.6	4,302,725	2,128,210	7,031,210	113.2	
Illinois	9,333,269	5,462,340	2,438,350	179.1	1,203,387	3,116,865	65.5	3,116,865	1,557,746	1,841,458	73.8	
Indiana	9,157,950	6,893,383	3,456,619	466.7	1,673,515	125.5	2,900,820	1,250,014	3,500,616	3,500,616	86.5	
Iowa	10,005,211	2,996,064	5,220,236	608.6	2,953,663	3,839,032	401.6	3,839,032	1,919,500	4,005,103	201.3	
Kentucky	6,984,271	2,255,539	990,365	150.7	495,182	1,465,770	28.0	1,465,770	732,884	4,630,418	41.7	
Louisiana	5,287,420	1,855,508	810,147	66.1	405,072	810,147	28.0	1,212,899	946,542	3,510,655	30.7	
Maine	3,299,867	925,151	838,968	58.6	419,646	731,560	21.7	731,560	365,780	1,556,090	23.6	
Maryland	3,094,808	958,513	1,049,363	58.6	524,646	448,610	17.5	448,610	224,305	2,345,857	6.3	
Massachusetts	5,255,300	333,935	4,322,180	3.1	2,161,090	487,090	20.3	487,090	243,545	2,683,697	2.3	
Michigan	11,562,296	9,222,035	4,958,669	332.3	2,468,431	3,133,800	123.7	3,133,800	1,566,275	2,962,633	56.9	
Minnesota	10,344,465	8,178,850	2,314,658	532.8	1,157,329	1,419,220	127.2	1,419,220	707,097	4,635,904	46.4	
Mississippi	6,635,344	4,101,546	913,040	448.9	456,820	3,589,490	55.9	3,589,490	1,693,940	4,484,584	161.1	
Missouri	11,479,090	3,918,264	7,309,623	392.7	3,642,893	257.2	2,980,011	1,486,262	4,306,276	3,506,276	113.1	
Montana	7,744,061	2,737,150	2,442,733	170.2	1,458,426	647,433	283.6	3,24,306	3,858,784	3,858,784	29.2	
Nebraska	7,809,353	1,571,360	2,905,687	272.2	1,458,426	988,954	139.3	1,490,577	494,458	4,934,147	139.3	
Nevada	4,821,864	1,571,360	567,128	27.2	487,083	91,600	11.5	91,600	8,300	2,970,945	.5	
New Hampshire	1,843,750	857,796	382,333	24.8	96,014	19,989	12.3	19,989	9,994	1,325,391	1.3	
New Jersey	5,094,295	2,466,551	2,251,052	34.7	1,021,251	19,989	28.9	19,989	9,994	2,799,945	28.8	
New Mexico	6,030,708	3,361,190	1,873,692	273.2	1,139,563	603,983	122.2	603,983	367,752	2,456,265	28.8	
New York	18,565,567	8,078,571	15,289,384	167.7	7,461,092	2,572,400	243.3	2,572,400	823,800	6,302,245	37.0	
North Carolina	8,877,837	2,488,072	3,506,143	313.0	1,746,172	2,139,840	245.2	2,139,840	954,770	4,934,147	72.8	
North Dakota	5,914,683	1,236,370	384,820	44.3	204,581	2,975,300	58.5	2,975,300	1,463,150	5,710,102	34.5	
Ohio	13,771,548	2,671,532	5,896,649	44.3	2,824,381	2,824,381	58.5	2,824,381	1,463,150	8,247,647	34.5	
Oklahoma	6,880,547	2,671,532	2,178,347	92.1	1,142,458	2,178,347	83.4	1,407,578	729,977	5,619,661	43.9	
Oregon	6,182,079	2,915,109	2,786,716	110.5	1,661,715	1,056,055	117.5	1,056,055	622,445	2,113,821	23.7	
Pennsylvania	16,129,804	6,656,250	7,678,677	111.3	3,850,511	4,117,693	105.5	4,117,693	1,385,452	6,993,026	61.6	
Rhode Island	1,843,750	213,888	678,336	3.4	339,168	173,727	5.6	173,727	86,864	1,310,774	2.1	
South Carolina	5,103,525	261,882	4,365,561	27.9	80,869	983,739	269.7	983,739	372,702	2,874,903	92.2	
South Dakota	6,162,747	1,384,652	777,493	188.6	772,488	423,492	36.5	423,492	423,492	4,880,893	57.4	
Tennessee	7,949,380	2,607,370	609,776	98.8	304,843	1,421,148	18.3	1,421,148	710,579	5,805,683	53.3	
Texas	21,506,431	12,178,430	7,632,063	685.4	3,772,158	5,033,235	453.7	5,033,235	2,428,394	11,176,977	339.9	
Utah	4,274,740	2,099,593	1,022,966	138.1	738,448	205,120	72.1	205,120	189,950	1,845,338	23.5	
Vermont	1,843,750	1,519,501	801,645	62.9	373,910	284,759	103.0	284,759	142,300	671,822	9.1	
Virginia	6,887,569	2,753,546	2,147,636	109.8	1,073,815	1,716,452	208.1	1,716,452	858,226	3,581,596	48.7	
Washington	5,907,615	3,882,942	2,020,071	149.9	1,060,771	517,744	84.7	517,744	270,660	2,538,757	3.8	
West Virginia	4,107,201	863,018	734,456	42.3	367,216	721,065	17.7	721,065	360,532	2,947,955	17.1	
Wisconsin	9,197,557	3,881,767	4,067,710	160.6	1,979,584	658,658	129.7	658,658	328,275	5,027,037	18.4	
Wyoming	4,722,322	2,814,018	1,466,308	394.0	898,816	379,966	152.6	379,966	234,004	1,857,156	34.2	
Hawaii	1,843,750	522,275	522,275	10.0	258,192	402,460	8.4	402,460	200,415	1,385,143	8.4	
Puerto Rico	625,000	-	-	-	-	-	-	-	-	625,000	-	
TOTALS	368,750,000	143,024,208	74,582,723	7,835.2	69,808,919	135,235,033	4,994.5	67,669,218	34,247,175	190,111,183	2,507.7	

¹ APPORTIONMENTS FOR FISCAL YEARS 1936 TO 1936, INCLUSIVE.

CURRENT STATUS OF UNITED STATES PUBLIC WORKS ROAD CONSTRUCTION

AS PROVIDED BY SECTION 204 OF THE NATIONAL INDUSTRIAL RECOVERY ACT (1934 FUNDS) AND BY THE ACT OF JUNE 18, 1934 (1935 FUNDS)

AS OF FEBRUARY 28, 1937

STATE	APPORTIONMENTS		COMPLETED			UNDER CONSTRUCTION					APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR NEW PROJECTS	
	Sec. 204 of the Act of June 18, 1934 (1934 Fund)	Act of June 18, 1934 (1935 Fund)	Total Cost	1934 Public Works Funds	1935 Public Works Funds	Mileage	Estimated Total Cost	1934 Public Works Funds	1935 Public Works Funds	Mileage	1934 Public Works Funds	1935 Public Works Funds	Mileage	1934 Public Works Funds	1935 Public Works Funds
Alabama	\$ 6,370,133	\$ 4,259,842	\$ 10,630,000	\$ 6,301,392	\$ 4,328,608	771.1	\$ 16,630,000	\$ 6,301,392	\$ 4,328,608	5.8	\$ 16,630,000	\$ 4,328,608	1.5	\$ 16,630,000	\$ 4,328,608
Arizona	5,211,930	2,641,935	7,853,865	5,203,675	2,650,190	543.2	7,853,865	5,203,675	2,650,190	6.5	7,853,865	2,650,190	10.9	7,853,865	2,650,190
Arkansas	6,744,335	3,428,049	10,172,384	6,687,424	3,484,960	619.9	10,172,384	6,687,424	3,484,960		10,172,384	3,484,960		10,172,384	3,484,960
California	15,607,394	7,932,806	23,540,200	15,607,394	7,932,806	798.7	23,540,200	15,607,394	7,932,806	.1	23,540,200	7,932,806		23,540,200	7,932,806
Colorado	6,874,530	3,486,006	10,360,536	6,874,530	3,486,006	633.7	10,360,536	6,874,530	3,486,006		10,360,536	3,486,006		10,360,536	3,486,006
Connecticut	2,665,140	1,454,868	4,120,008	2,719,018	1,401,000	74.0	4,120,008	2,719,018	1,401,000		4,120,008	1,401,000		4,120,008	1,401,000
Delaware	1,819,088	923,335	2,742,423	1,819,088	923,335	190.0	2,742,423	1,819,088	923,335		2,742,423	923,335		2,742,423	923,335
Florida	5,231,834	2,661,345	7,893,179	5,231,834	2,661,345	366.4	7,893,179	5,231,834	2,661,345	3.5	7,893,179	2,661,345	.3	7,893,179	2,661,345
Georgia	10,091,185	5,113,491	15,204,676	10,091,185	5,113,491	294.2	15,204,676	10,091,185	5,113,491	73.4	15,204,676	5,113,491	7.8	15,204,676	5,113,491
Idaho	4,486,249	2,877,486	7,363,735	4,486,249	2,877,486	501.4	7,363,735	4,486,249	2,877,486	1.1	7,363,735	2,877,486	7.5	7,363,735	2,877,486
Illinois	17,570,770	8,921,401	26,492,171	17,570,770	8,921,401	715.6	26,492,171	17,570,770	8,921,401	2.4	26,492,171	8,921,401		26,492,171	8,921,401
Indiana	10,031,843	5,088,963	15,120,806	10,031,843	5,088,963	482.7	15,120,806	10,031,843	5,088,963	5.0	15,120,806	5,088,963	.1	15,120,806	5,088,963
Iowa	10,059,660	5,118,361	15,178,021	10,059,660	5,118,361	1,222.3	15,178,021	10,059,660	5,118,361	13.1	15,178,021	5,118,361		15,178,021	5,118,361
Kansas	10,093,664	5,117,875	15,211,539	10,093,664	5,117,875	1,312.3	15,211,539	10,093,664	5,117,875	2.8	15,211,539	5,117,875		15,211,539	5,117,875
Kentucky	7,537,599	3,818,311	11,355,910	7,537,599	3,818,311	812.3	11,355,910	7,537,599	3,818,311		11,355,910	3,818,311		11,355,910	3,818,311
Louisiana	5,828,591	2,963,932	8,792,523	5,828,591	2,963,932	259.5	8,792,523	5,828,591	2,963,932	11.5	8,792,523	2,963,932	7.3	8,792,523	2,963,932
Maine	3,369,917	1,711,586	5,081,503	3,369,917	1,711,586	131.5	5,081,503	3,369,917	1,711,586	1.4	5,081,503	1,711,586		5,081,503	1,711,586
Maryland	3,564,527	1,810,058	5,374,585	3,564,527	1,810,058	152.2	5,374,585	3,564,527	1,810,058	1.5	5,374,585	1,810,058		5,374,585	1,810,058
Massachusetts	6,397,100	3,350,474	9,747,574	6,397,100	3,350,474	115.5	9,747,574	6,397,100	3,350,474	7.2	9,747,574	3,350,474		9,747,574	3,350,474
Michigan	12,136,227	6,452,568	18,588,795	12,136,227	6,452,568	768.1	18,588,795	12,136,227	6,452,568		18,588,795	6,452,568		18,588,795	6,452,568
Minnesota	10,950,595	5,425,591	16,376,186	10,950,595	5,425,591	813.9	16,376,186	10,950,595	5,425,591		16,376,186	5,425,591		16,376,186	5,425,591
Mississippi	6,978,675	3,540,227	10,518,902	6,978,675	3,540,227	723.8	10,518,902	6,978,675	3,540,227	21.4	10,518,902	3,540,227	.7	10,518,902	3,540,227
Missouri	12,180,306	6,173,740	18,354,046	12,180,306	6,173,740	1,441.2	18,354,046	12,180,306	6,173,740	6.4	18,354,046	6,173,740		18,354,046	6,173,740
Montana	7,439,734	3,769,734	11,209,468	7,439,734	3,769,734	1,098.8	11,209,468	7,439,734	3,769,734		11,209,468	3,769,734		11,209,468	3,769,734
Nebraska	7,828,961	3,964,364	11,793,325	7,828,961	3,964,364	1,018.4	11,793,325	7,828,961	3,964,364	27.5	11,793,325	3,964,364	5.9	11,793,325	3,964,364
Nevada	4,545,917	2,302,336	6,848,253	4,545,917	2,302,336	78.8	6,848,253	4,545,917	2,302,336		6,848,253	2,302,336		6,848,253	2,302,336
New Hampshire	1,909,835	959,462	2,869,297	1,909,835	959,462	78.3	2,869,297	1,909,835	959,462		2,869,297	959,462		2,869,297	959,462
New Jersey	6,346,039	3,200,879	9,546,918	6,346,039	3,200,879	85.9	9,546,918	6,346,039	3,200,879		9,546,918	3,200,879		9,546,918	3,200,879
New Mexico	5,792,935	2,941,700	8,734,635	5,792,935	2,941,700	216.2	8,734,635	5,792,935	2,941,700	10.3	8,734,635	2,941,700	.6	8,734,635	2,941,700
New York	22,330,101	11,327,921	33,658,022	22,330,101	11,327,921	821.3	33,658,022	22,330,101	11,327,921	3.9	33,658,022	11,327,921	.1	33,658,022	11,327,921
North Carolina	9,522,293	4,840,941	14,363,234	9,522,293	4,840,941	1,345.8	14,363,234	9,522,293	4,840,941	21.5	14,363,234	4,840,941	28.0	14,363,234	4,840,941
North Dakota	5,804,448	2,938,967	8,743,415	5,804,448	2,938,967	2,186.9	8,743,415	5,804,448	2,938,967	16.3	8,743,415	2,938,967	4.2	8,743,415	2,938,967
Ohio	15,484,892	7,865,012	23,349,904	15,484,892	7,865,012	792.1	23,349,904	15,484,892	7,865,012	4.0	23,349,904	7,865,012		23,349,904	7,865,012
Oklahoma	9,216,798	4,685,180	13,901,978	9,216,798	4,685,180	805.5	13,901,978	9,216,798	4,685,180	1.5	13,901,978	4,685,180	.7	13,901,978	4,685,180
Oregon	6,106,896	3,097,814	9,204,710	6,106,896	3,097,814	468.0	9,204,710	6,106,896	3,097,814	3.4	9,204,710	3,097,814	3.7	9,204,710	3,097,814
Pennsylvania	18,891,004	9,590,788	28,481,792	18,891,004	9,590,788	1,094.1	28,481,792	18,891,004	9,590,788		28,481,792	9,590,788		28,481,792	9,590,788
Rhode Island	1,996,708	1,014,572	3,011,280	1,996,708	1,014,572	89.1	3,011,280	1,996,708	1,014,572	9.4	3,011,280	1,014,572	1.7	3,011,280	1,014,572
South Carolina	5,469,165	2,770,954	8,240,119	5,469,165	2,770,954	618.5	8,240,119	5,469,165	2,770,954	29.5	8,240,119	2,770,954	6.1	8,240,119	2,770,954
South Dakota	6,011,479	3,047,643	9,059,122	6,011,479	3,047,643	1,571.6	9,059,122	6,011,479	3,047,643		9,059,122	3,047,643		9,059,122	3,047,643
Tennessee	8,462,619	4,308,951	12,771,570	8,462,619	4,308,951	1,462.7	12,771,570	8,462,619	4,308,951	3.6	12,771,570	4,308,951	5.9	12,771,570	4,308,951
Texas	24,284,619	12,591,521	36,876,140	24,284,619	12,591,521	2,760.0	36,876,140	24,284,619	12,591,521	2.0	36,876,140	12,591,521	8.0	36,876,140	12,591,521
Utah	4,194,708	2,132,691	6,327,399	4,194,708	2,132,691	590.9	6,327,399	4,194,708	2,132,691		6,327,399	2,132,691		6,327,399	2,132,691
Vermont	1,867,573	948,007	2,815,580	1,867,573	948,007	141.0	2,815,580	1,867,573	948,007	30.5	2,815,580	948,007	5.2	2,815,580	948,007
Virginia	7,416,757	3,765,387	11,182,144	7,416,757	3,765,387	614.0	11,182,144	7,416,757	3,765,387		11,182,144	3,765,387		11,182,144	3,765,387
Washington	6,115,867	3,106,442	9,222,309	6,115,867	3,106,442	302.7	9,222,309	6,115,867	3,106,442		9,222,309	3,106,442		9,222,309	3,106,442
West Virginia	4,474,234	2,280,335	6,754,569	4,474,234	2,280,335	212.9	6,754,569	4,474,234	2,280,335	9.4	6,754,569	2,280,335	.7	6,754,569	2,280,335
Wisconsin	5,724,861	2,891,531	8,616,392	5,724,861	2,891,531	619.7	8,616,392	5,724,861	2,891,531		8,616,392	2,891,531		8,616,392	2,891,531
Wyoming	4,501,527	2,287,172	6,788,699	4,501,527	2,287,172	1,037.1	6,788,699	4,501,527	2,287,172	.3	6,788,699	2,287,172	2.5	6,788,699	2,287,172
District of Columbia	1,918,469	973,842	2,892,311	1,918,469	973,842	22.3	2,892,311	1,918,469	973,842		2,892,311	973,842		2,892,311	973,842
Hawaii	1,871,062	949,778	2,820,840	1,871,062	949,778	51.1	2,820,840	1,871,062	949,778		2,820,840	949,778		2,820,840	949,778
TOTALS	394,000,000	200,000,000	594,000,000	394,000,000	200,000,000	35,056.0	594,000,000	394,000,000	200,000,000	399.8	594,000,000	200,000,000	110.6	594,000,000	200,000,000

